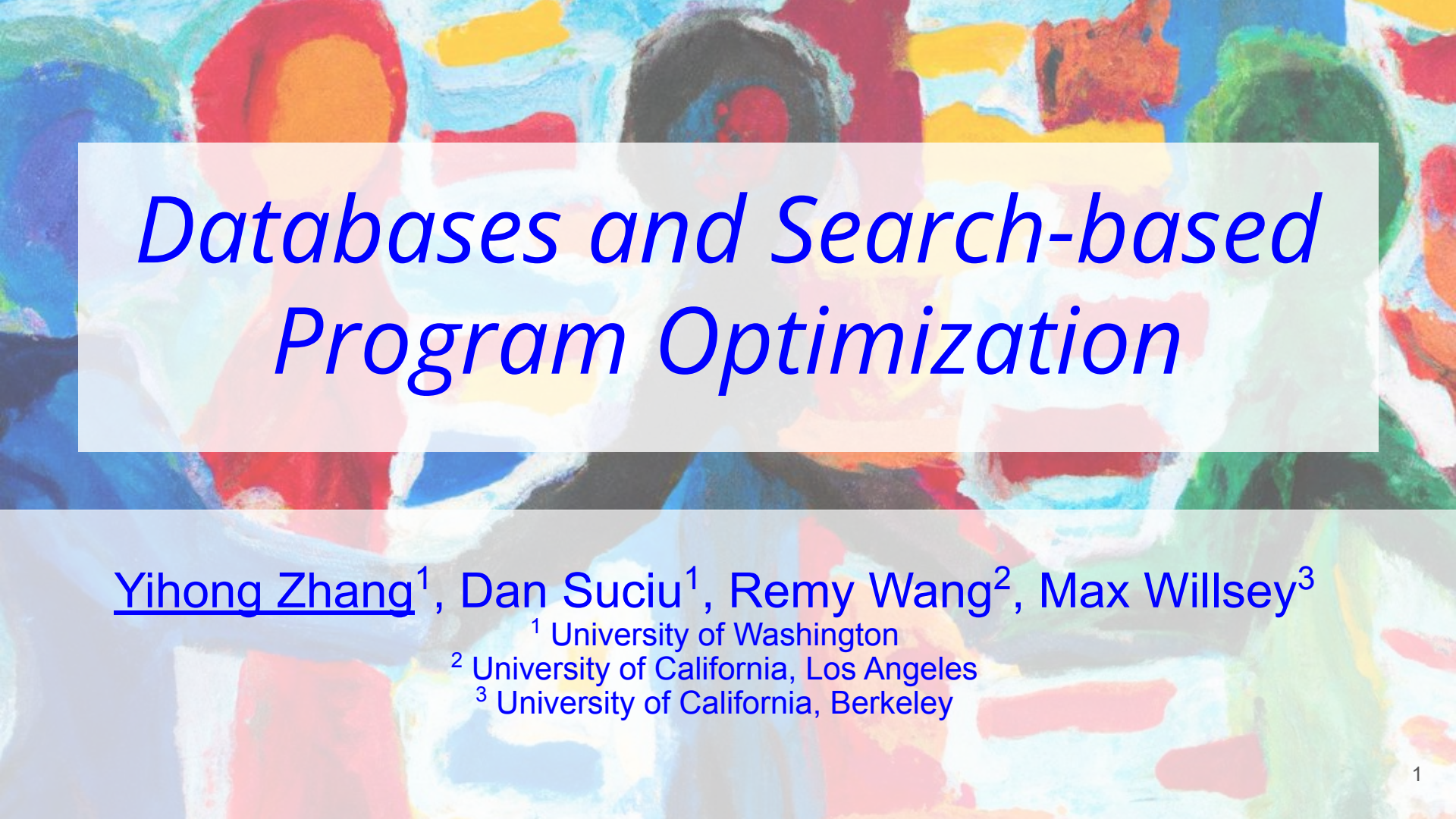


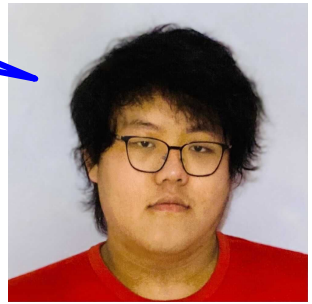
Databases and Search-based Program Optimization

Yihong Zhang[1], Dan Suciu[1], Remy Wang[2], Max Willsey[3]
[1] University of Washington
[2] University of California, Los Angeles
[3] University of California, Berkeley

About me

About me

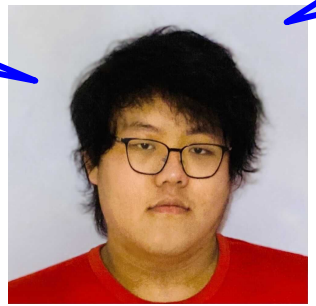
PhD student at
Univ. of Washington



About me

PhD student at
Univ. of Washington

I work on
Equality Saturation



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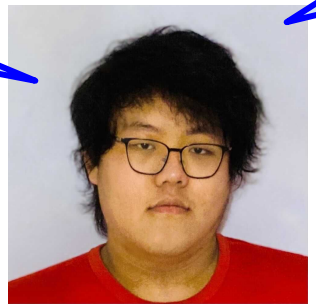

I work on
Equality Saturation

Program optimization technique used in 50+ projects

- Awards: PLDI ‘15, OOPSLA ‘21, ASPLOS ‘24, POPL ‘24.
- Industry users: Intel, Certora, Bytecode Alliance, …
- EqSat Papers: VLDB ‘20, SIGMOD ‘22 ‘23, ICDE ‘22, PLDI ‘20 ‘24^{x2}, OOPSLA ‘21 ‘23 ‘24, ASPLOS ‘21 ‘23 ‘24^{x3} ‘25, POPL ‘09 ‘23, ICFP ‘24, CCA ‘21, CCS ‘22, CGO ‘24^{x2} ‘25^{x2}, DAC ‘23^{x2} ‘24, EGRAPHS ‘22 ‘23^{x4}, FCCM ‘22^{x2}, PACT ‘22^{x2} ‘24, DAC ‘23^{x2} 24, FMCAD ‘22, MLSys ‘21, MAPS ‘21, IDDM ‘23, SIGA ‘19, TOG ‘22 …

About me

PhD student at
Univ. of Washington

I work on
Equality Saturation

The more I study EqSat,
the more I realize:
It's just databases!

Program optimization technique used in 50+ projects

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Optimizing programs using term rewriting

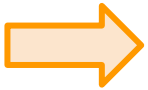
(a * 2) / 2  a

Optimizing programs using term rewriting

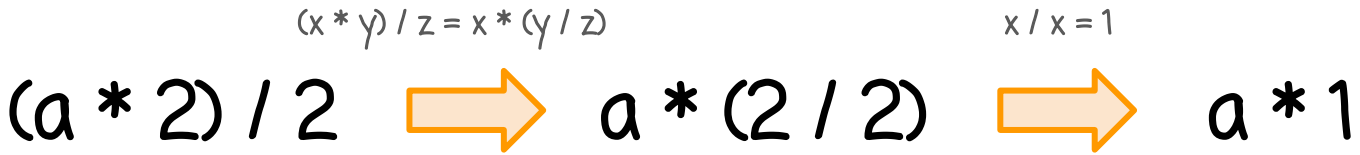
```
(a * 2) / 2
```

Optimizing programs using term rewriting

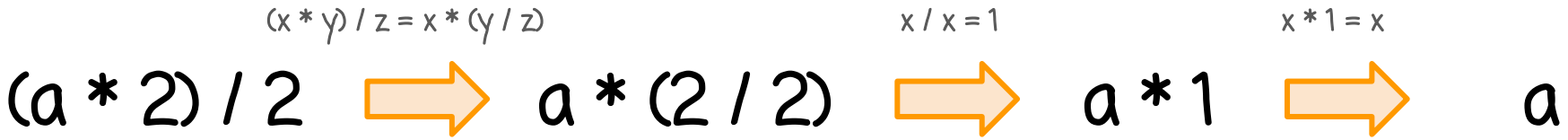
$(x * y) / z = x * (y / z)$

$(a * 2) / 2$  $a * (2 / 2)$

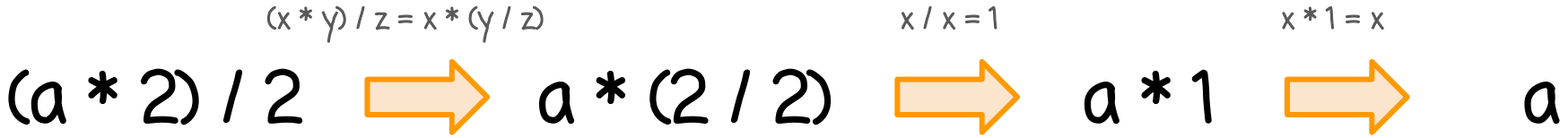
Optimizing programs using term rewriting



Optimizing programs using term rewriting

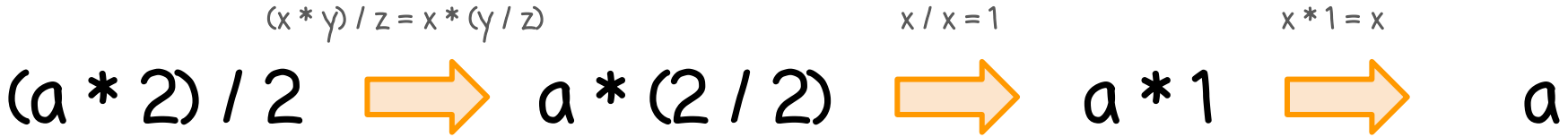


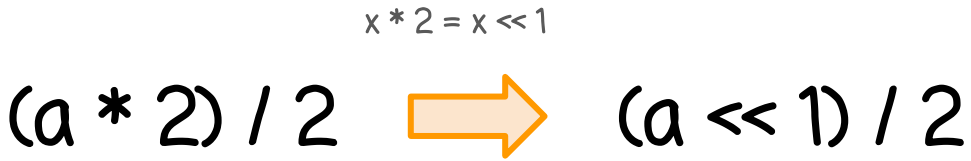
Optimizing programs using term rewriting



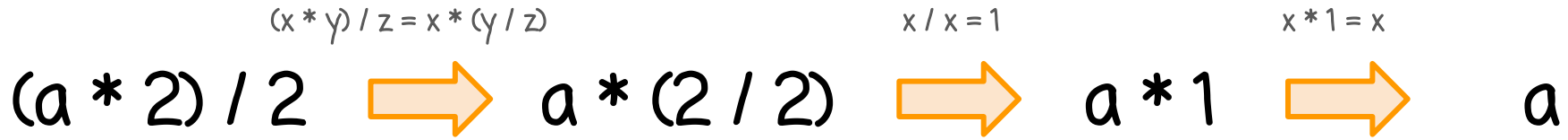
(a * 2) / 2

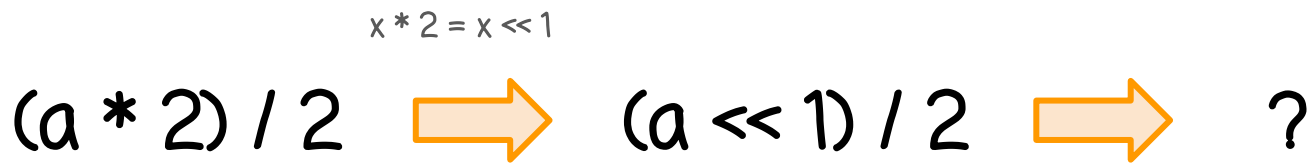
Optimizing programs using term rewriting





Optimizing programs using term rewriting





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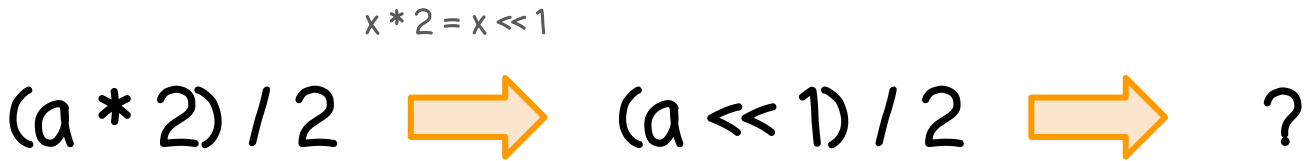
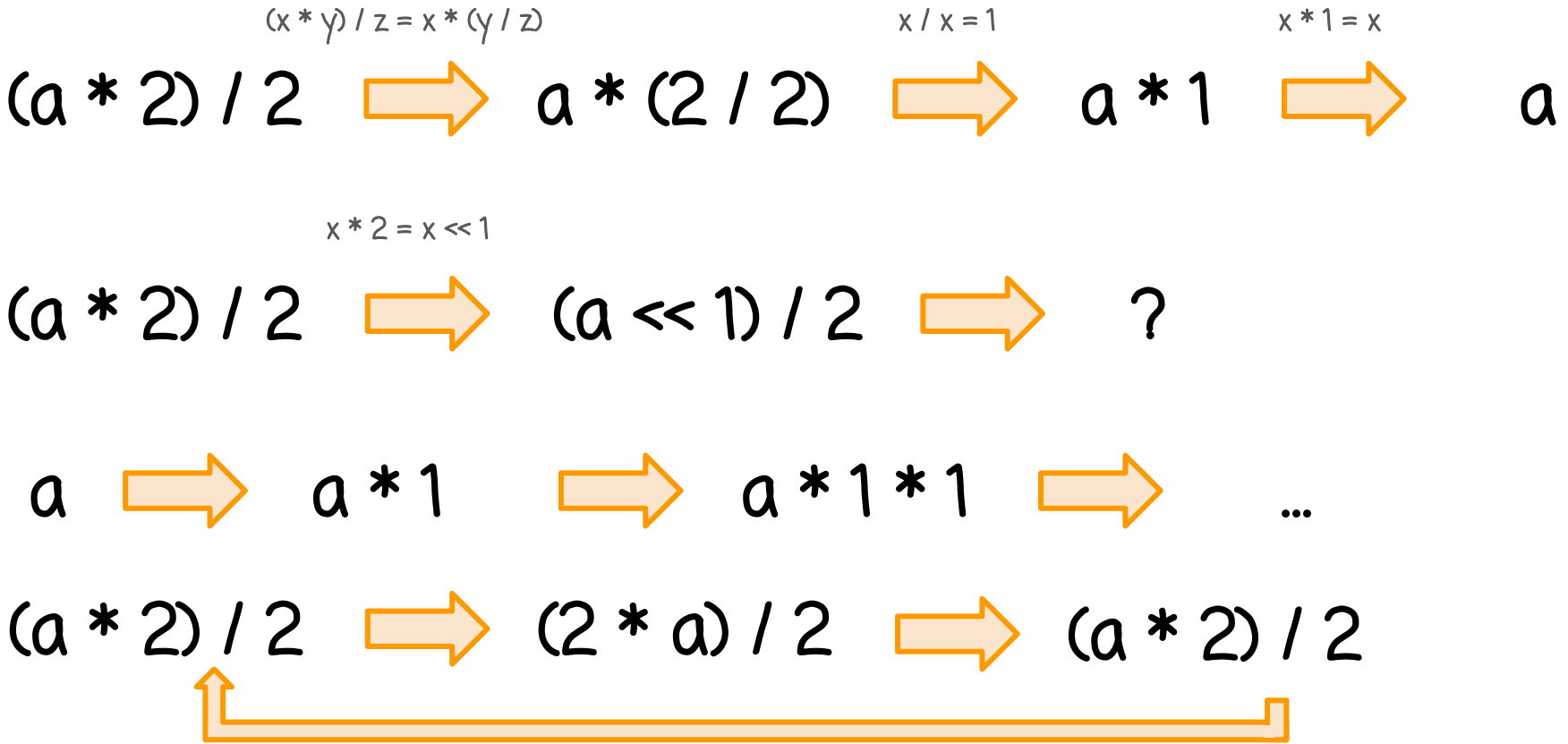
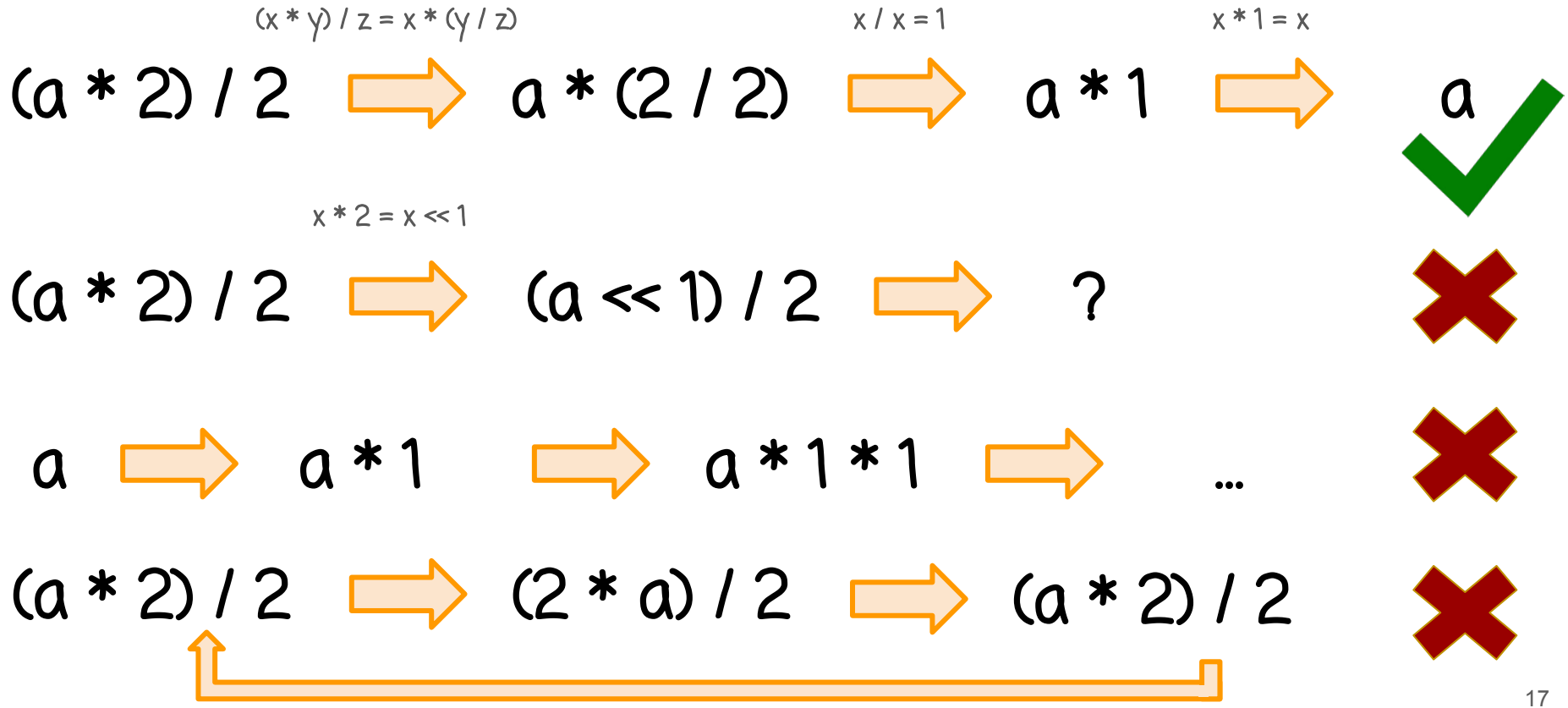


Optimizing programs using term rewriting



Optimizing programs using term rewriting



Optimizing programs using term rewriting

$(x * y) / z = x * (y / z)$ $\quad x / x = 1$ $\quad x * 1 = x$

$(a * 2) / 2 \Rightarrow a * (2 / 2) \Rightarrow a * 1 \Rightarrow a$ ✓

$x * 2 = x << 1$

$(a * 2) / 2 \Rightarrow (a << 1) / 2 \Rightarrow ?$ ✗

$a \Rightarrow a * 1 \Rightarrow a * 1 * 1 \Rightarrow \ldots$ ✗

$(a * 2) / 2 \Rightarrow (2 * a) / 2 \Rightarrow (a * 2) / 2$

Optimizing programs using term rewriting

$(x * y) / z = x * (y / z)$ $x / x = 1$ $x * 1 = x$

$(a * 2) / 2 \Rightarrow a * (2 / 2) \Rightarrow a * 1 \Rightarrow a$ ✓

$x * 2 = x << 1$

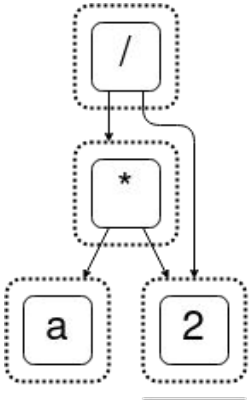
(a ✗

Equality Saturation: apply all the rules all the time!

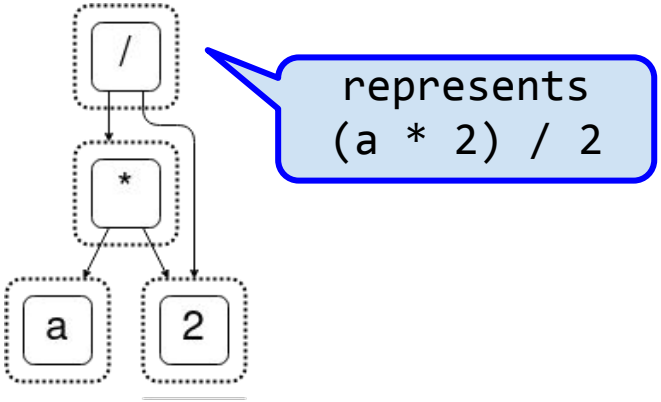
$a \Rightarrow a * 1 \Rightarrow a * 1 * 1 \Rightarrow \ldots$ ✗

$(a * 2) / 2 \Rightarrow (2 * a) / 2 \Rightarrow (a * 2) / 2$

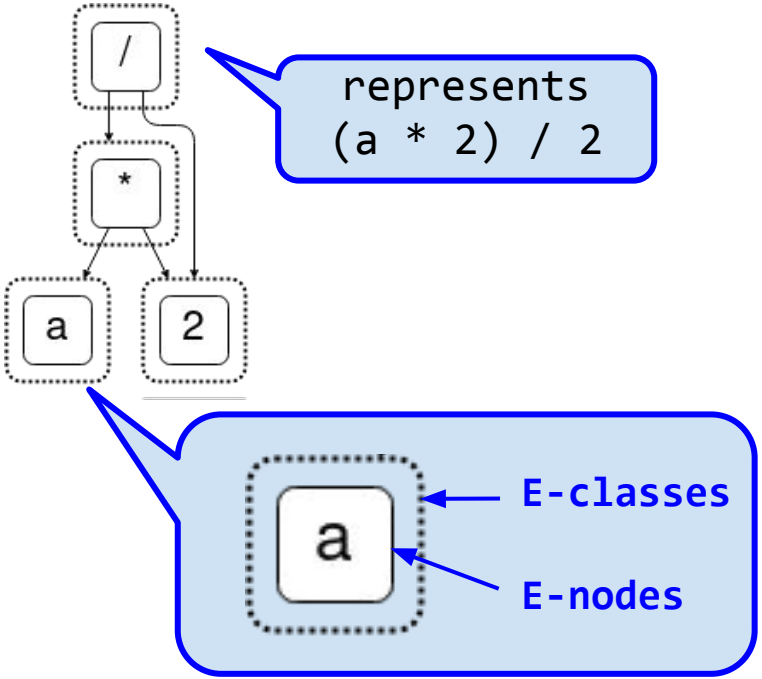
E-graphs and Equality saturation



E-graphs and Equality saturation



E-graphs and Equality saturation



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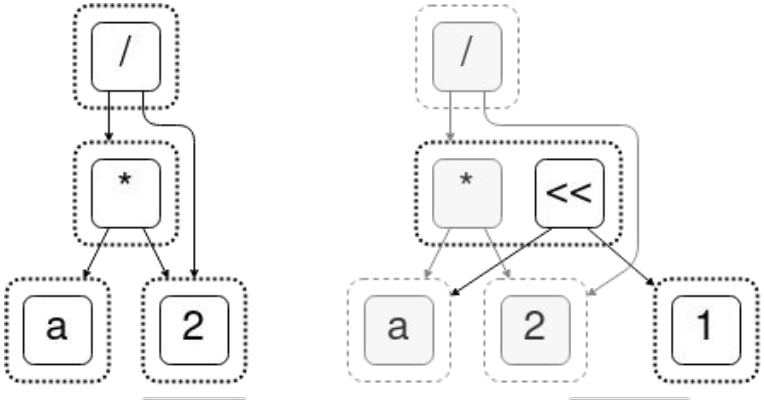
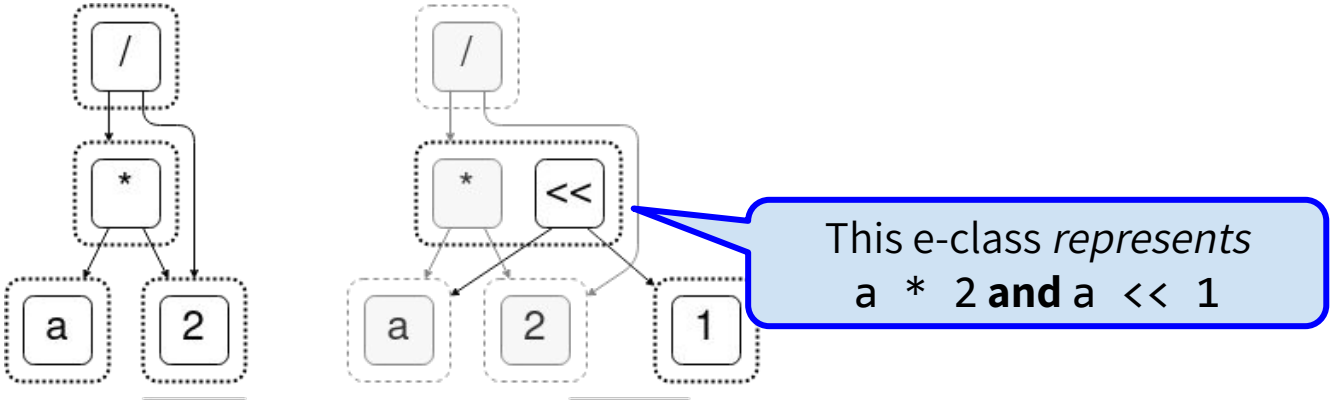


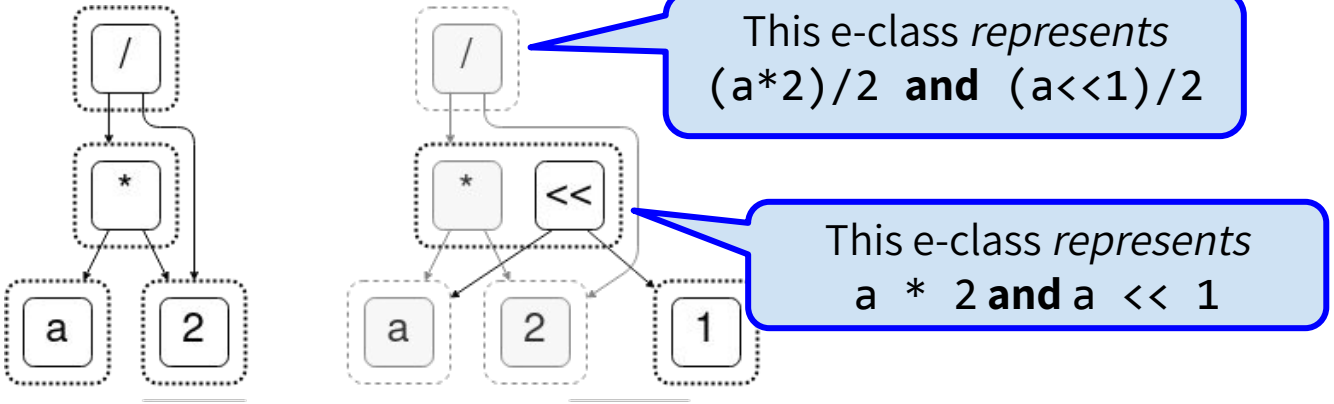
x * 2 => x << 1

E-graphs and Equality saturation



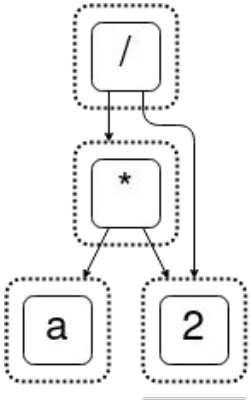

```
x * 2 => x << 1
```

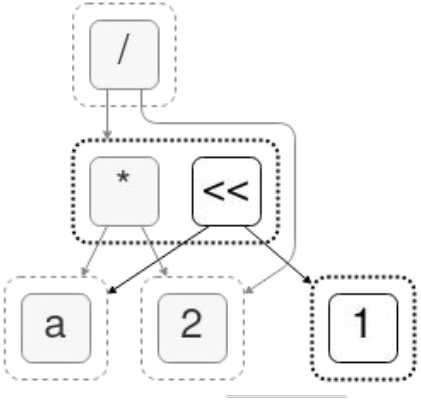
E-graphs and Equality saturation

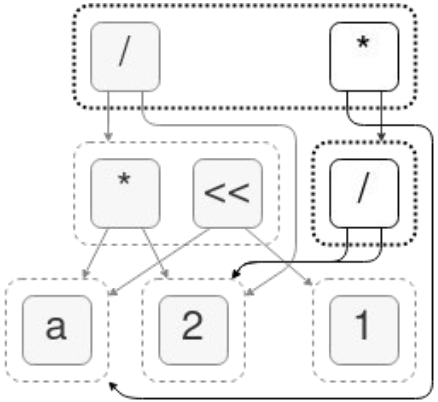


`x * 2 => x << 1`

E-graphs and Equality saturation



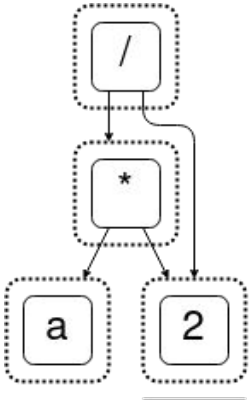


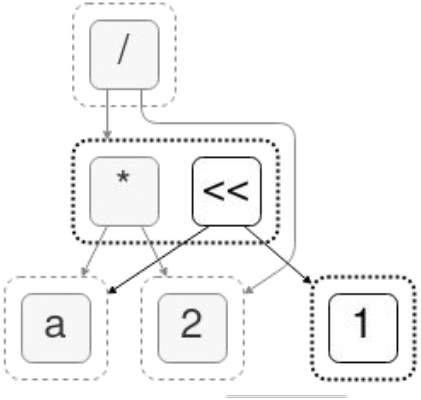


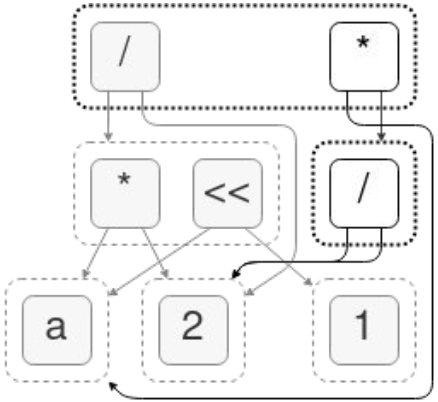
`x * 2 => x << 1`

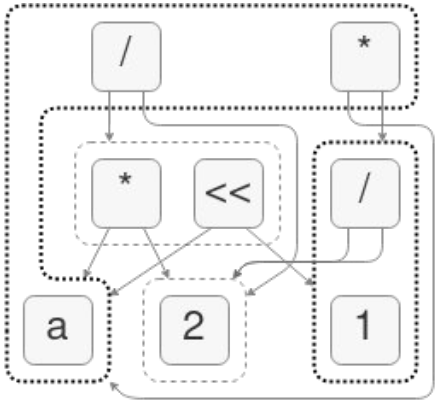
`(x * y) / z => x * (y / z)`

E-graphs and Equality saturation









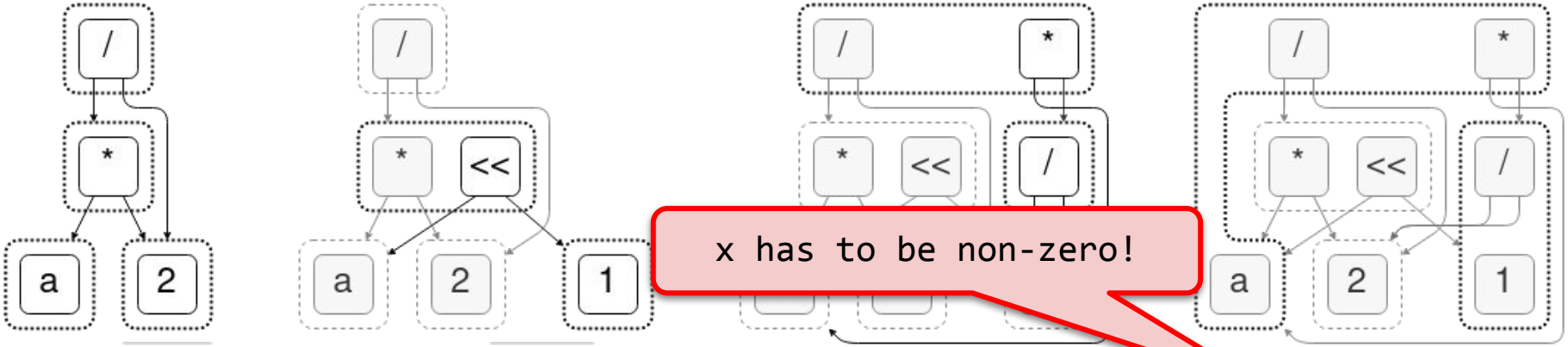
`x * 2 => x << 1`

`(x * y) / z => x * (y / z)`

`x / x => 1`
`x * 1 => x`

loop until
fixpoint / timeout!

E-graphs and Equality saturation



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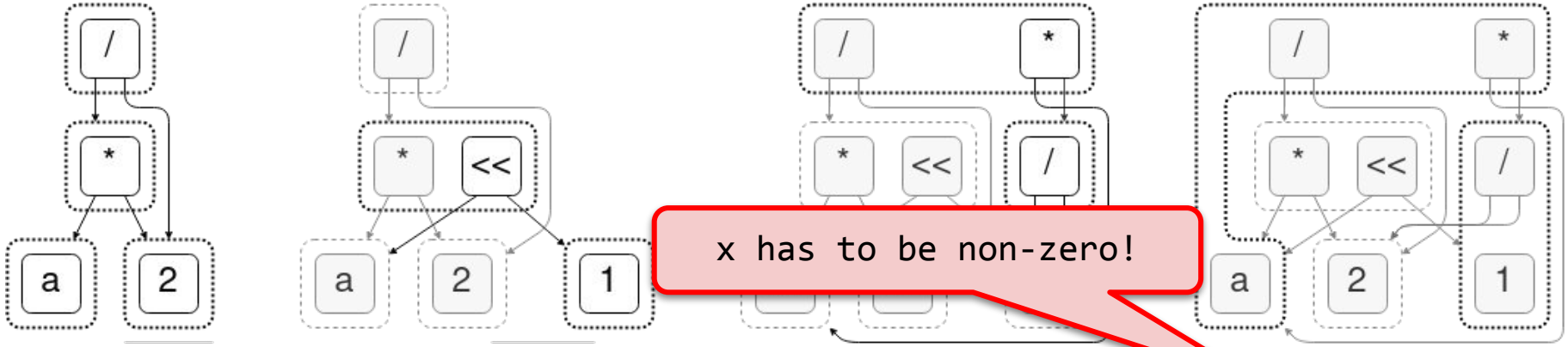

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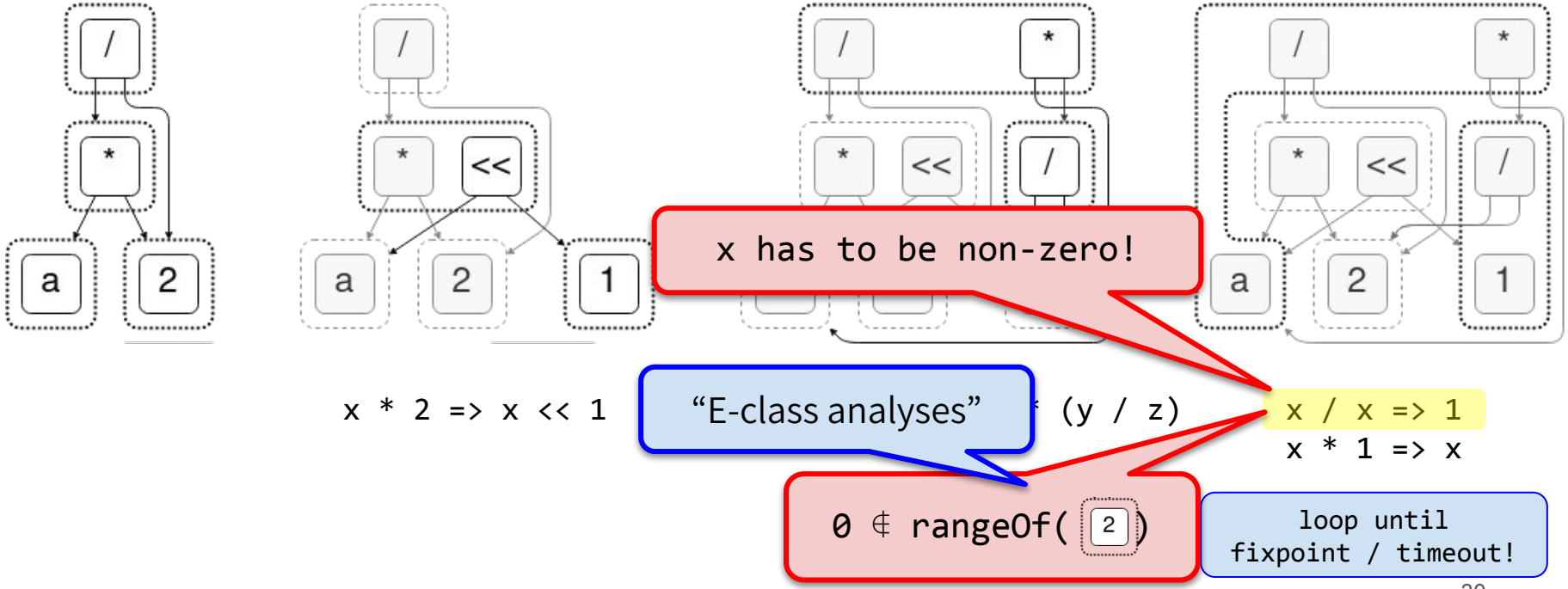

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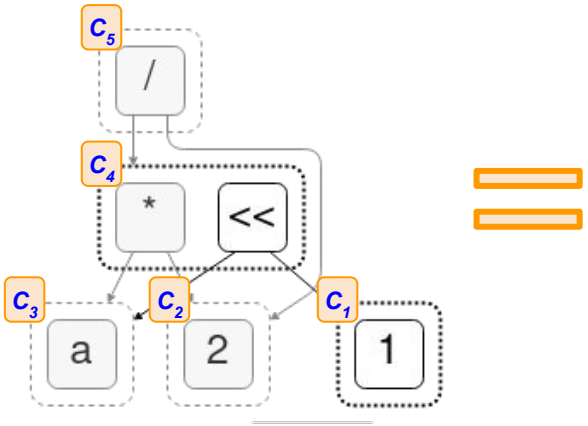

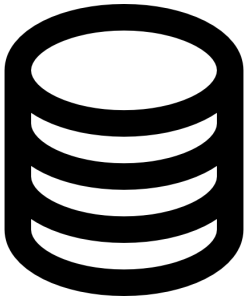

```
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```

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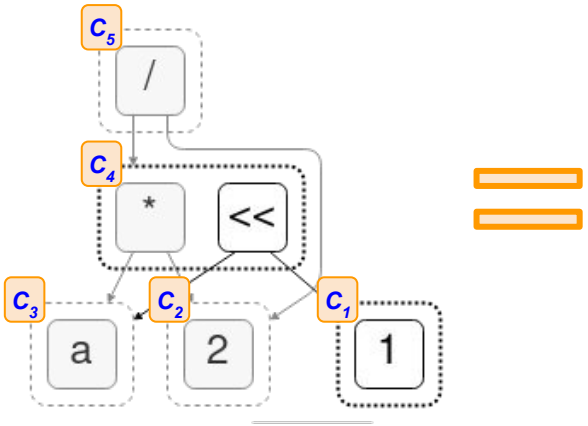


=

Table: number

child	out
1	c_1
2	c_2

Table: variable

child	out
"a"	c_3

Table: *

ch_1	ch_2	out
c_3	c_2	c_4

Table: <<

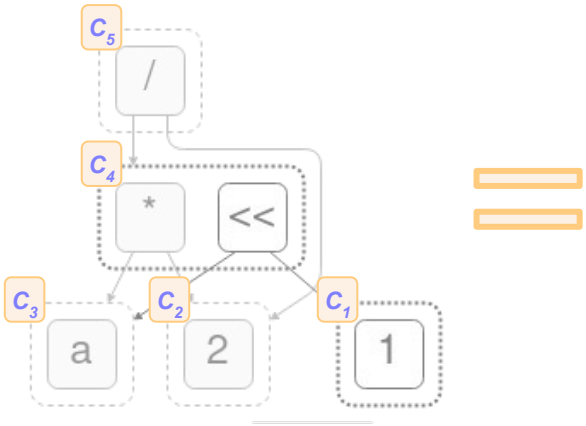
ch_1	ch_2	out
c_3	c_1	c_4

Table: /

ch_1	ch_2	out
c_4	c_2	c_5

`x * 2 => x << 1`

E-graphs and Equality saturation



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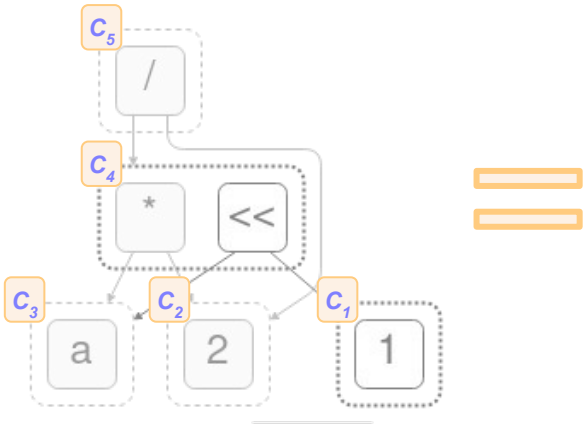

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Table: /

ch_1	ch_2	out
c_4	c_2	c_5

```
x * 2 => x << 1
```

```
<<(x, c1, r) :- *(x, c2, r)
```

E-graphs and Equality saturation

E-graphs		Databases
E-nodes		Tuples
E-classes		Labeled null
Rewrite rules		Tuple-generating dep. (TGD)
Congruence closure		Functional dep. (FD)
E-class analyses E-graph extraction		Monotonic Aggregation
Equality Saturation		**Chase**

E-graphs and Equality saturation

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E-class analyses E-graph extraction		Monotonic Aggregation
Equality Saturation		**Chase**

More details in our ICDT paper (Suciu, Wang, Zhang)

It’s also true the other way around!

Volcano/Cascades is also EqSat!

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Volcano and Cascades are query optimization frameworks that combines rule-based optimization and cost-based optimization.

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Volcano and Cascades are query optimization frameworks that combines rule-based optimization and cost-based optimization.

Key ideas

- apply rewrite rules over a memo table data structure
- use a cost model to select the best query plan

Volcano/Cascades is also EqSat!

Volcano and Cascades are query optimization frameworks that combines rule-based optimization and cost-based optimization.

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E-graph

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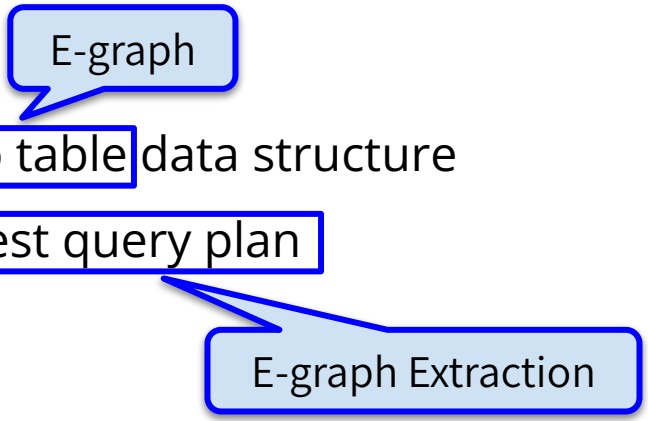

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Key ideas

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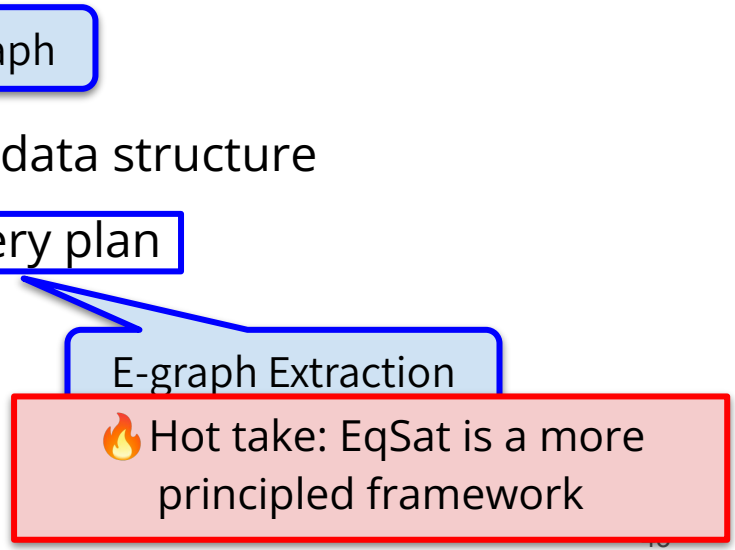
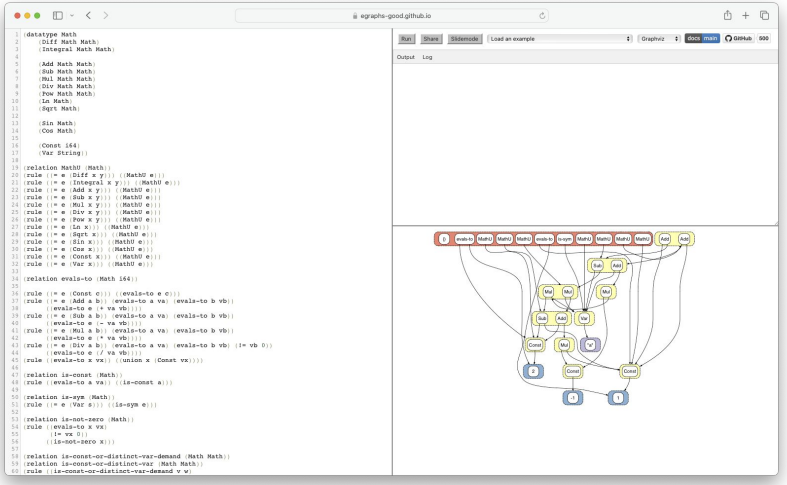


Case study:
Rule- and cost-based optimization of matrix expressions

https://github.com/egraphs-good/egglog/tree/icdt-db-x-demo



```
(sort Mat)
(function Matrix (String String String) Mat)
(function Prod (Mat Mat) Mat)
(function Agg (String Mat) Mat)
```

```
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(function Matrix (String String String) Mat)
(function Prod (Mat Mat) Mat)
(function Agg (String Mat) Mat)
```

Table: Matrix

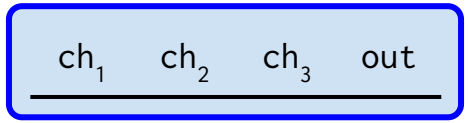


Table: Prod

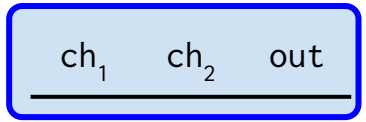
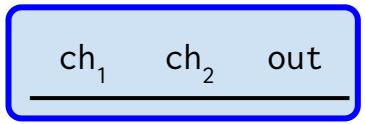


Table: Agg



```
(sort Mat)
(function Matrix (String String String) Mat)
(function Prod (Mat Mat) Mat)
(function Agg (String Mat) Mat)

(let A (Matrix "A" "i" "j")) ;; A_{i,j}
(let B (Matrix "B" "j" "k")) ;; B_{j,k}
(let C (Matrix "C" "k" "l")) ;; C_{k,l}
;; Σ_k Σ_j (AB)C
(let ABC (Agg "k" (Agg "j"
           (Prod (Prod A B) C))))
```

Table: Matrix

ch_1	ch_2	ch_3	out

Table: Prod

ch_1	ch_2	out

Table: Agg

ch_1	ch_2	out

```
(sort Mat)
(function Matrix (String String String) Mat)
(function Prod (Mat Mat) Mat)
(function Agg (String Mat) Mat)

(let A (Matrix "A" "i" "j")) ;; A_{i,j}
(let B (Matrix "B" "j" "k")) ;; B_{j,k}
(let C (Matrix "C" "k" "l")) ;; C_{k,l}
;; Σ_k Σ_j (AB)C
(let ABC (Agg "k" (Agg "j"
            (Prod (Prod A B) C))))
```

Table: Matrix

ch_1	ch_2	ch_3	out
“A”	“i”	“j”	C_A
“B”	“j”	“k”	C_B
“C”	“k”	“l”	C_C

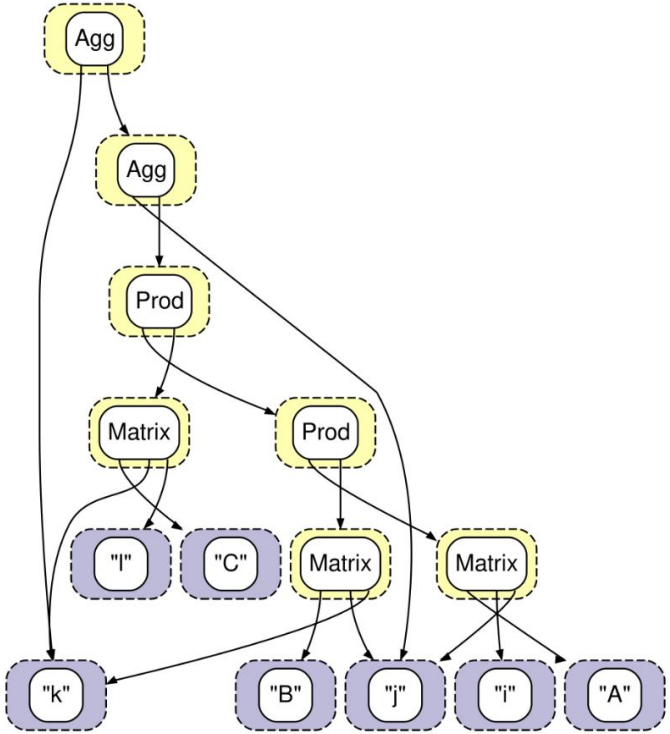
Table: Prod

ch_1	ch_2	out
C_A	C_B	C_{AB}
C_{AB}	C_C	C_{ABC}

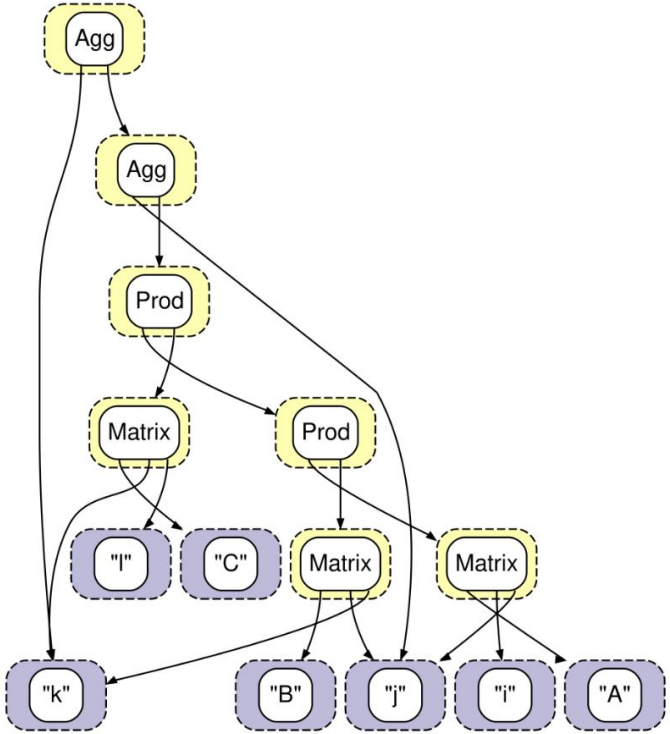
Table: Agg

ch_1	ch_2	out
“j”	C_{ABC}	C_j
“k”	C_j	C_k

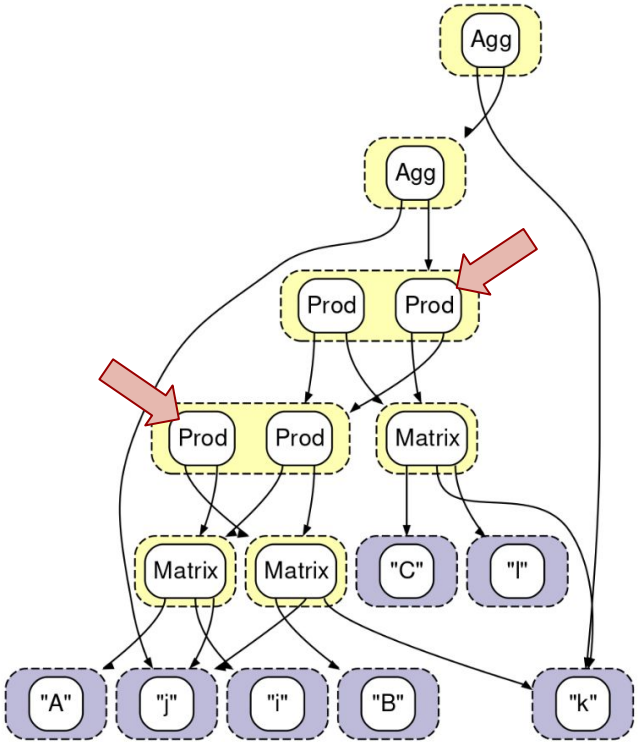
```
(sort Mat)
(function Matrix (String String String) Mat)
(function Prod (Mat Mat) Mat)
(function Agg (String Mat) Mat)

(let A (Matrix "A" "i" "j")) ;; A_{i,j}
(let B (Matrix "B" "j" "k")) ;; B_{j,k}
(let C (Matrix "C" "k" "l")) ;; C_{k,l}
;; Σ_k Σ_j (AB)C
(let ABC (Agg "k" (Agg "j"
           (Prod (Prod A B) C))))
```



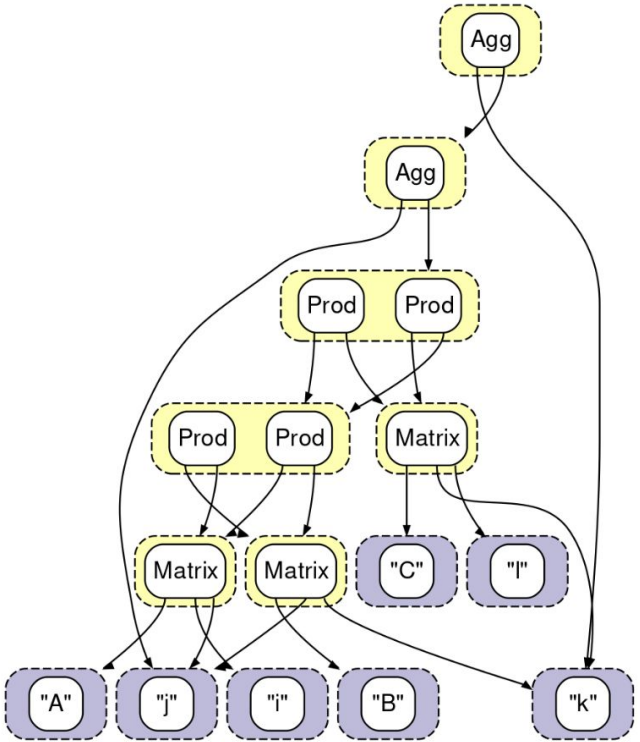
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
```



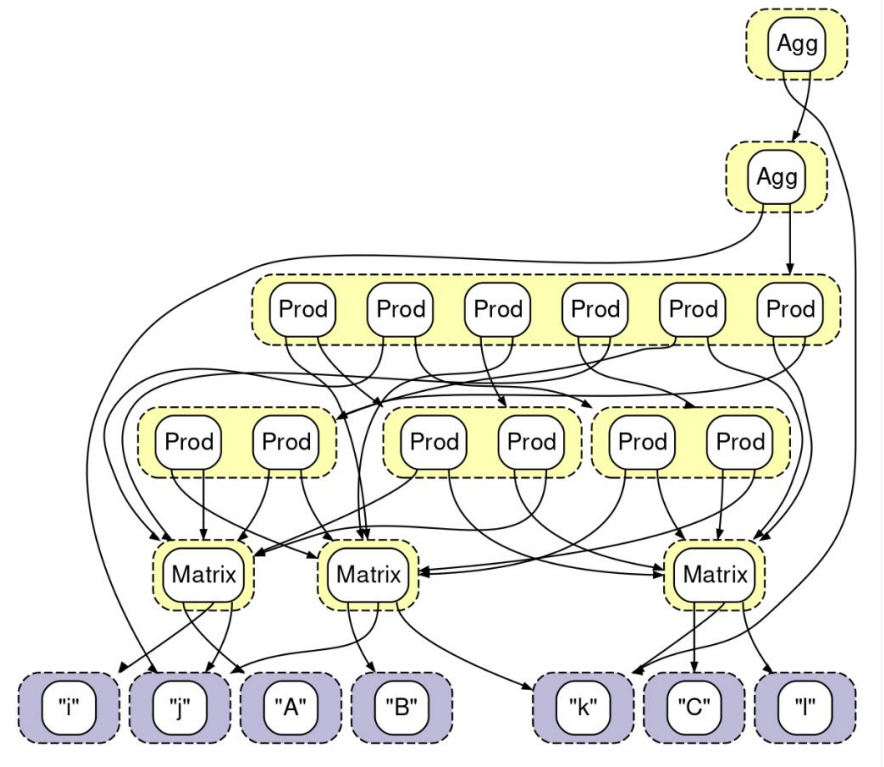
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
```



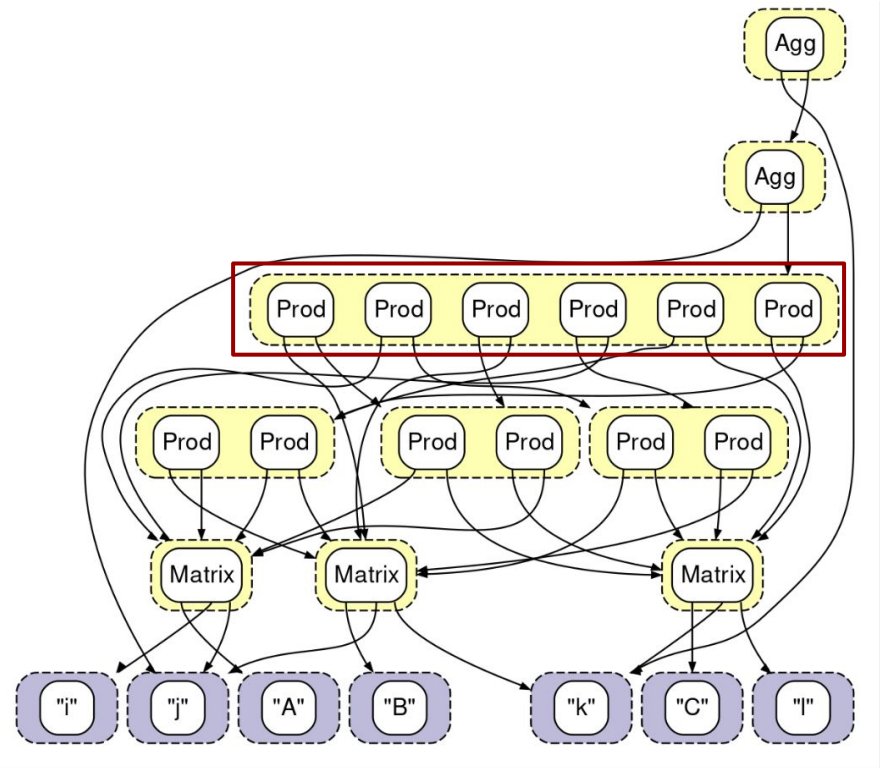
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
```



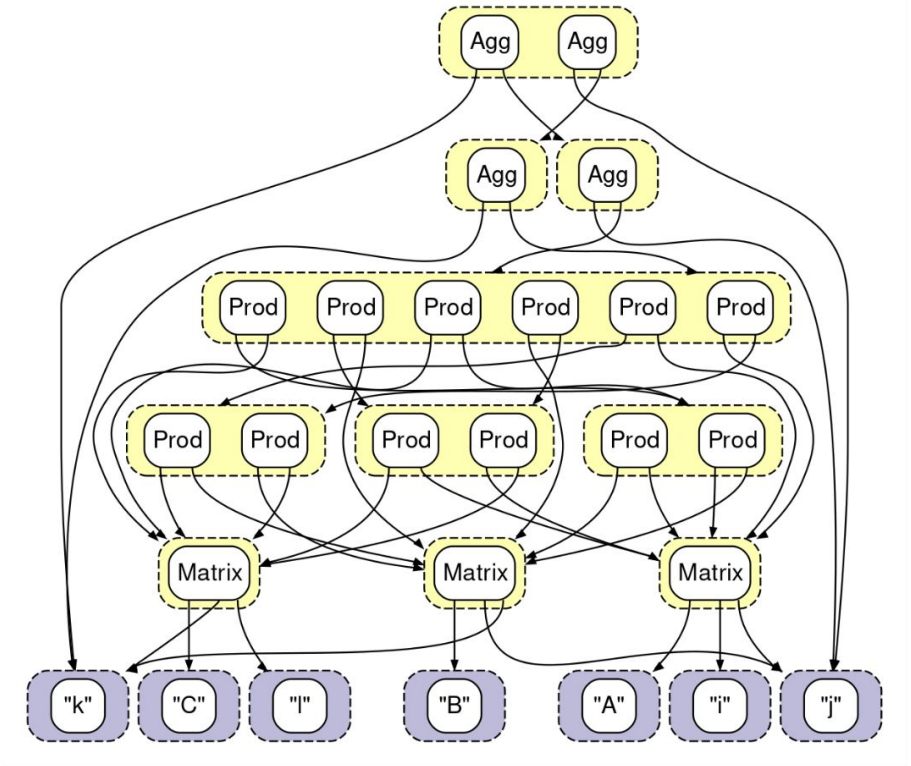
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
```



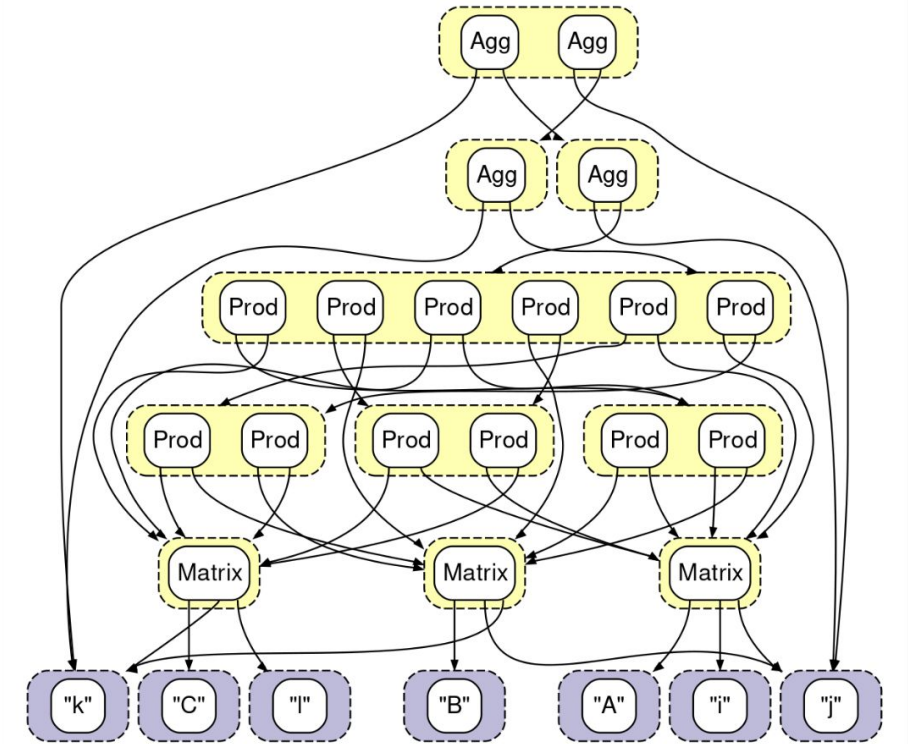
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
```



```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
```

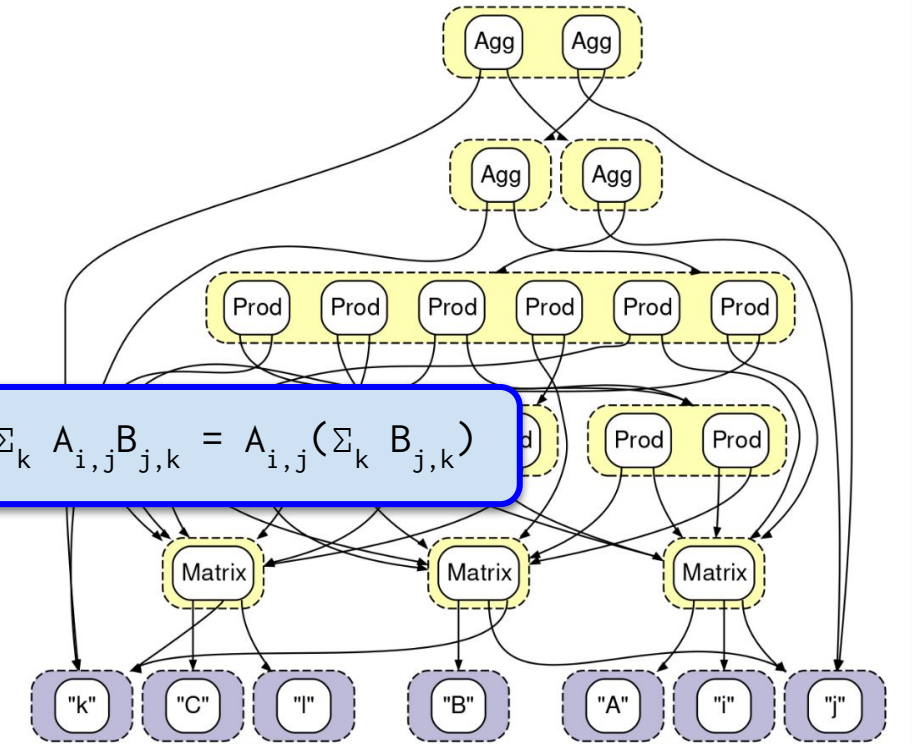


```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((∉ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((∉ v (vars-of x)))
```

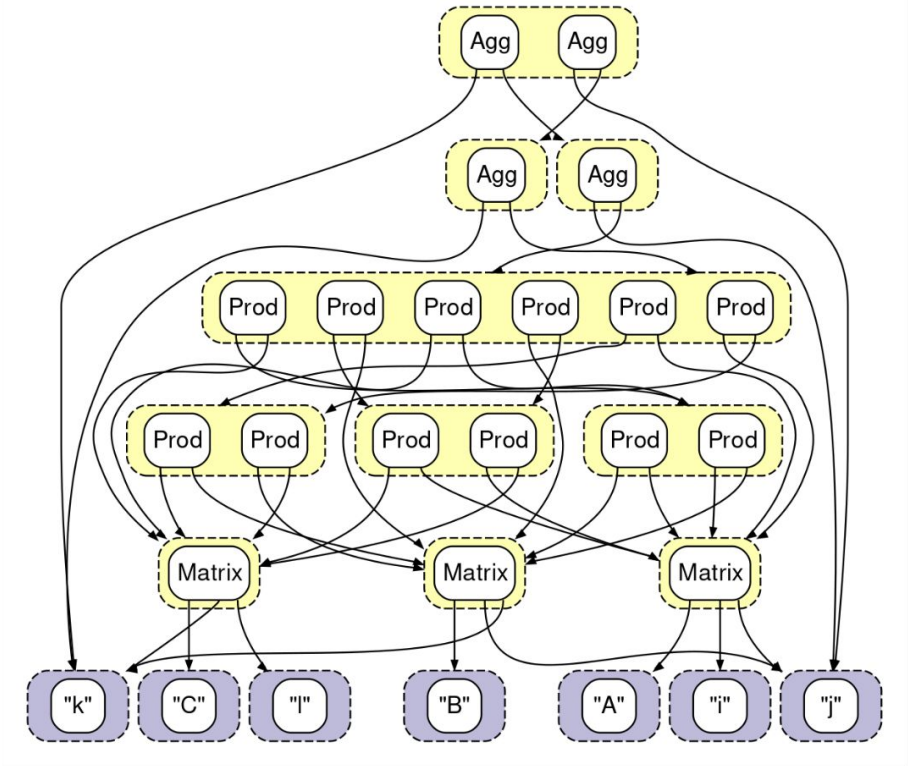


```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((∉ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((∉ v (vars-of x)))
```

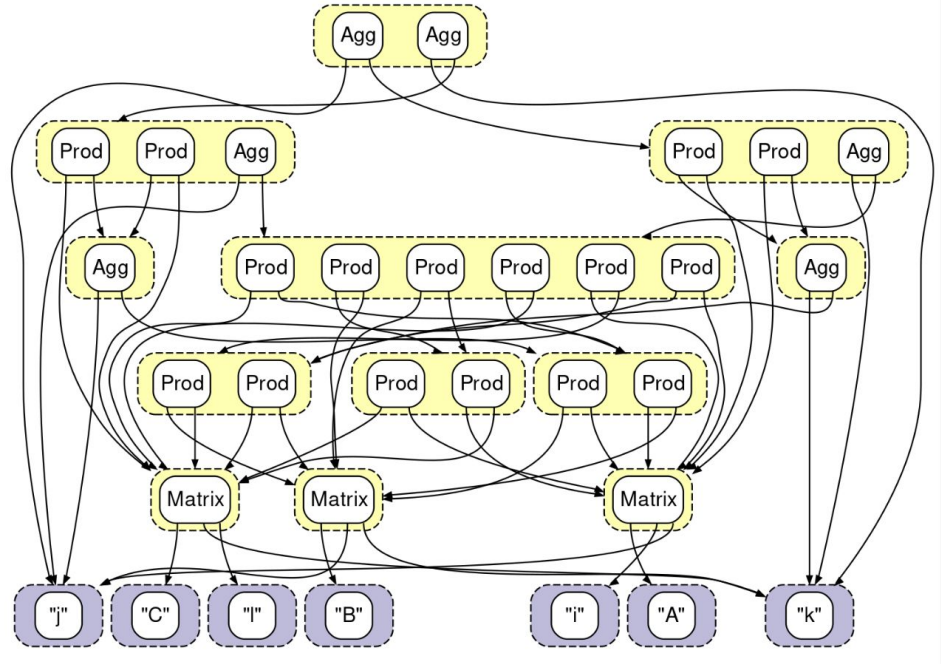
Example: $\Sigma_k A_{i,j} B_{j,k} = A_{i,j} (\Sigma_k B_{j,k})$



```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((∉ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((∉ v (vars-of x)))
```

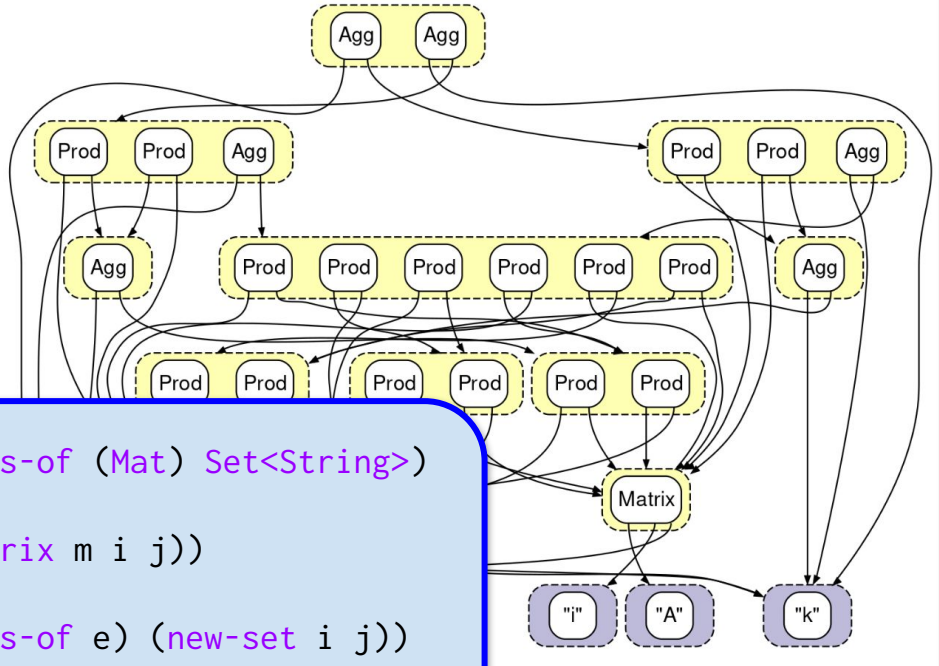


```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((∉ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((∉ v (vars-of x)))
```



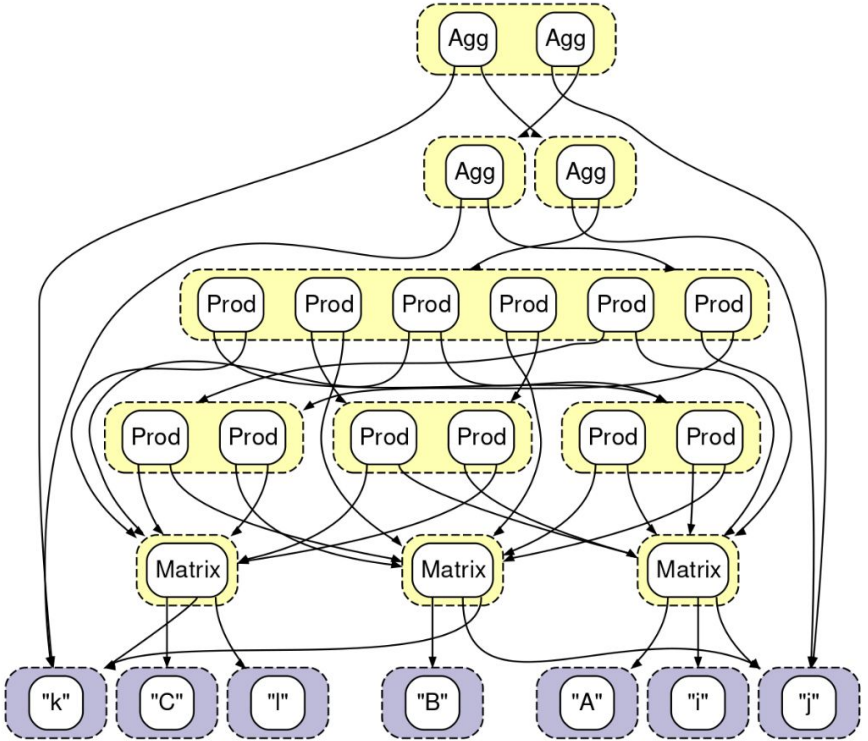
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((∉ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((∉ v (vars-of x))))
```

```
(function vars-of (Mat) Set<String>)
(rule (
    (= e (Matrix m i j))
) (
    (set (vars-of e) (new-set i j))
))
                ...
```



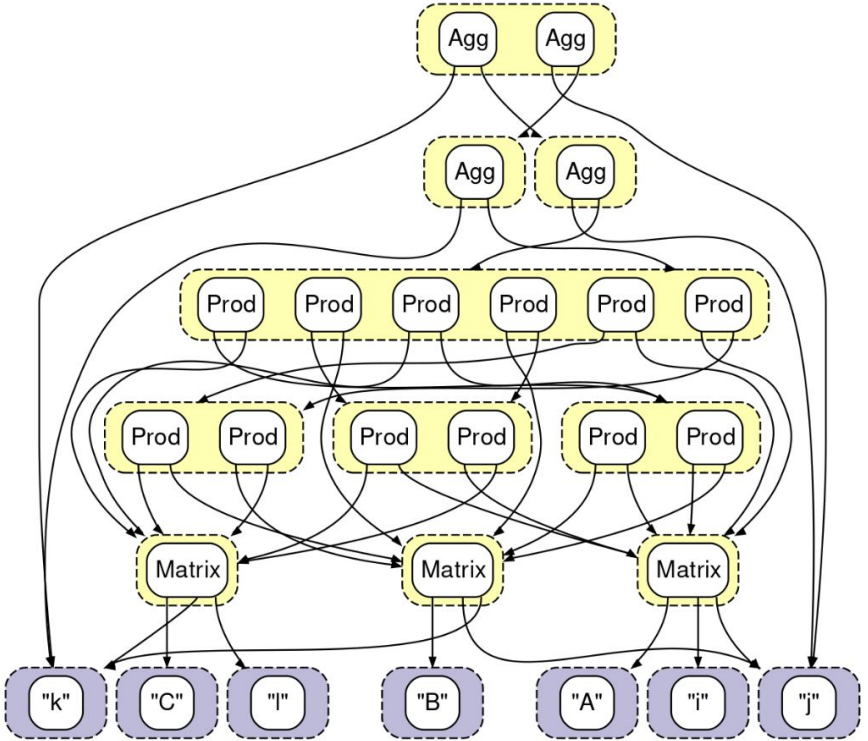
```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((¢ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((¢ v (vars-of x))))
```

Rule-based Opt. ✓

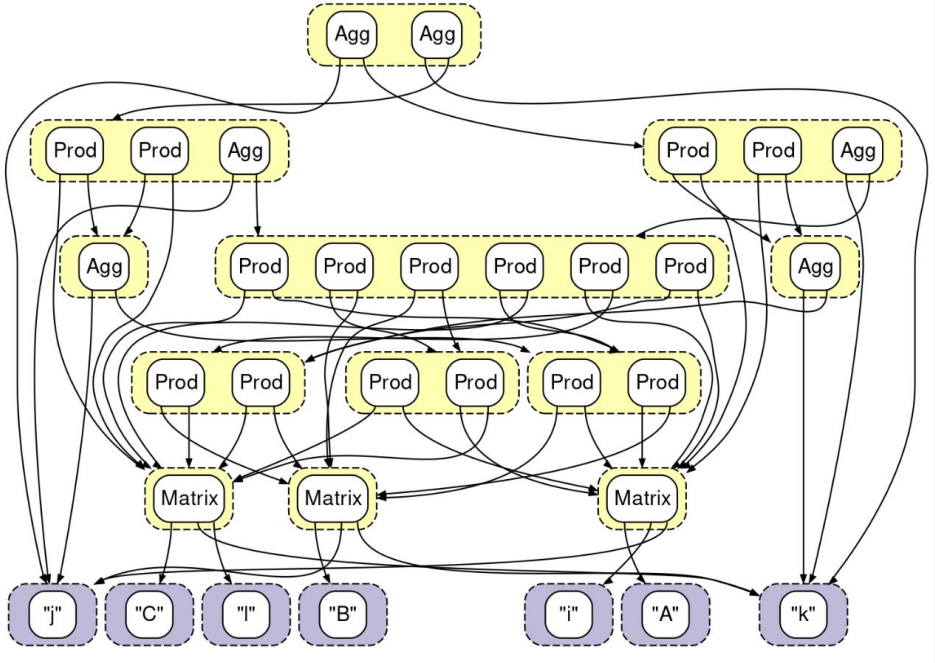


```
;; commutativity
(rewrite (Prod x y) (Prod y x))
;; associativity
(rewrite (Prod (Prod x y) z)
         (Prod x (Prod y z)))
;; commuting aggregation
(rewrite (Agg v1 (Agg v2 e))
         (Agg v2 (Agg v1 e)))
;; pushing down aggregation
(rewrite (Agg v (Prod x y))
         (Prod (Agg v x) y)
  :when ((¢ v (vars-of y))))
(rewrite (Agg v (Prod x y))
         (Prod x (Agg v y))
  :when ((¢ v (vars-of x))))
```

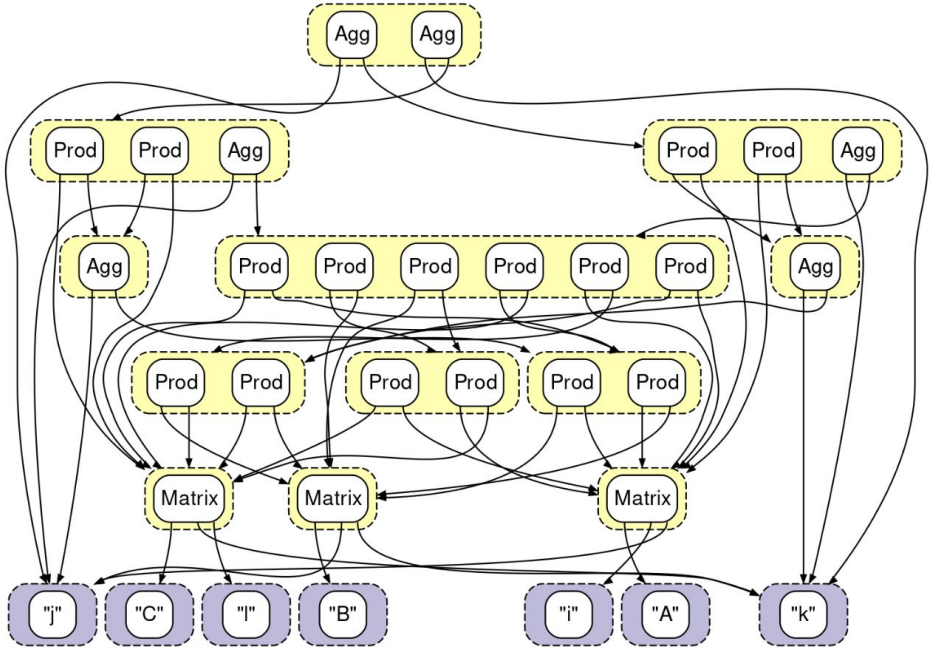
Cost-based Opt. ?



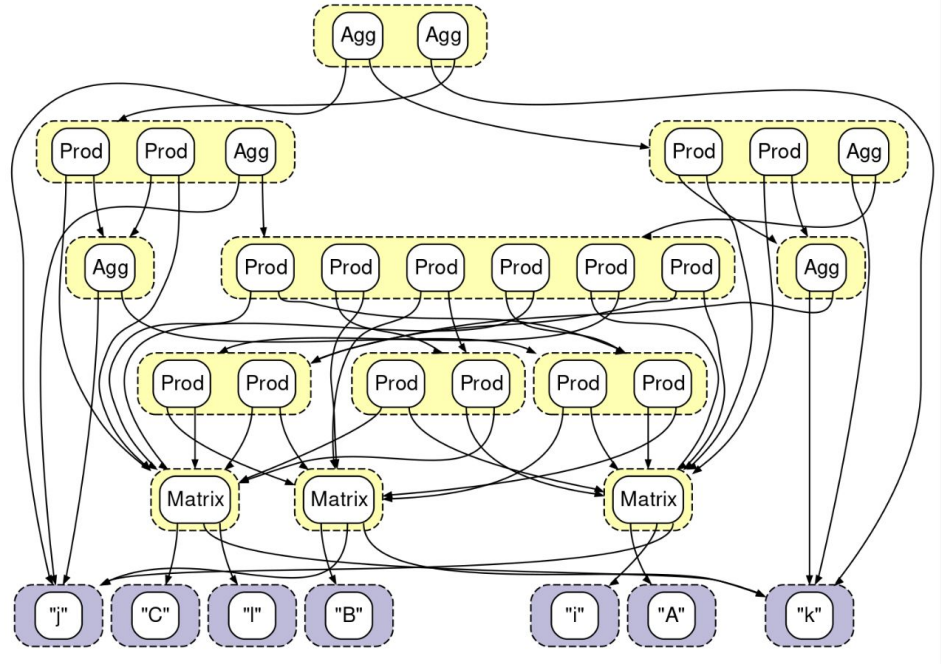
```
;; user provided dimension information
(function dim-of (String) i64)

;; estimate the size of a matrix expr
(function size-of (Mat) i64)
(rule (
    (= (vars-of e) vs)
) (
    (set (size-of e)
         (Π (map dim-of vs)))
))
```



```
;; user provided dimension information
(function dim-of (String) i64)

;; estimate the size of a matrix expr
(function size-of (Mat) i64)
(rule (
    (= (vars-of e) vs)
) (
    (set (size-of e)
         (Π (map dim-of vs)))
))

;; set the cost of an expr as its size
(rule (
    (= (size-of (Prod e1 e2)) k)
) (
    (set-cost (Prod e1 e2) k)
))
…
```

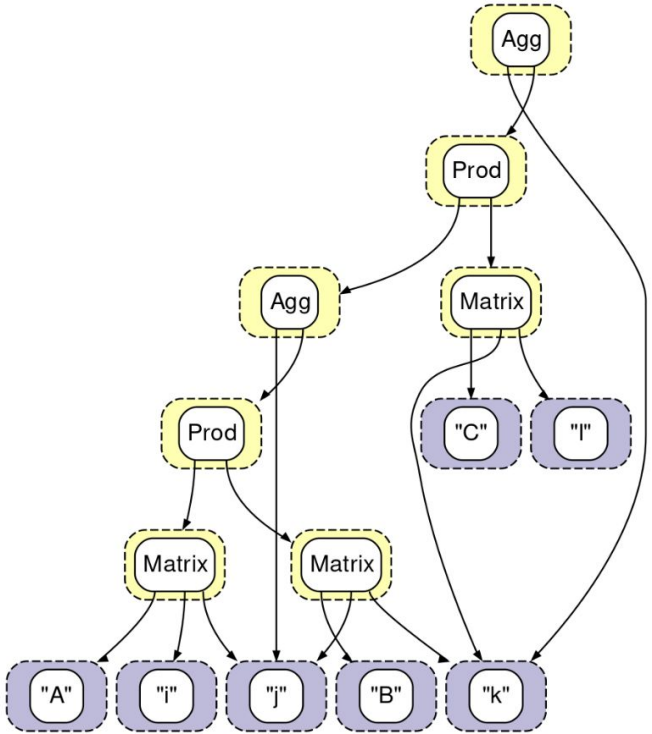


```
(set (dim-of "i") 256)
(set (dim-of "j") 64)
(set (dim-of "k") 16)
(set (dim-of "l") 256)
```



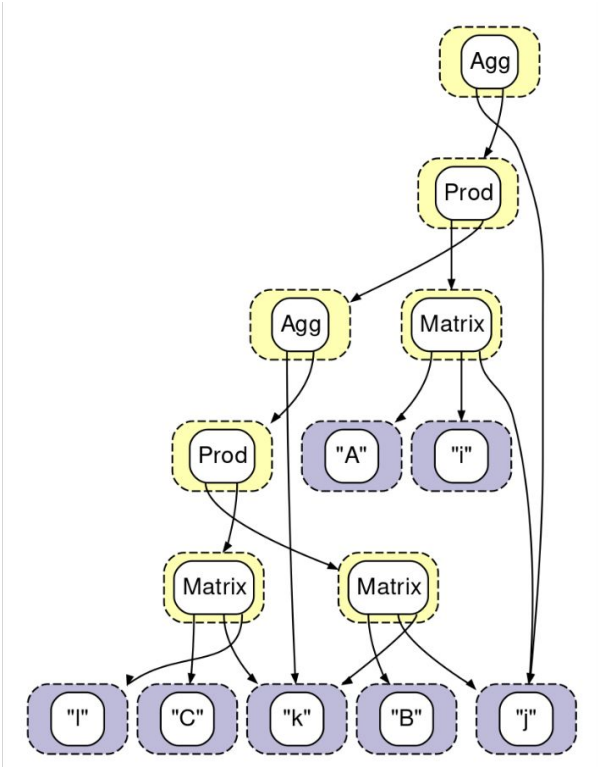
```
(set (dim-of "i") 256)
(set (dim-of "j") 64)
(set (dim-of "k") 16)
(set (dim-of "l") 256)

(extract ABC)
```

```
extracted with cost 1401867:
(Agg "k" (Prod
  (Agg "j" (Prod
    (Matrix "A" "i" "j")
    (Matrix "B" "j" "k")))
  (Matrix "C" "k" "l")))
```



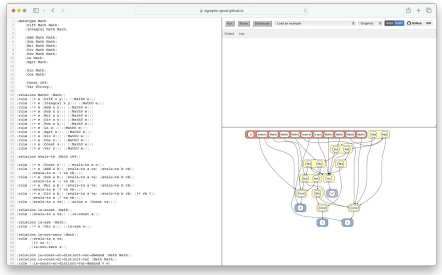
```
(set (dim-of "i") 256)
(set (dim-of "j") 64)
(set (dim-of "k") ~~16~~128)
(set (dim-of "l") 256)

(extract ABC)
```

```
extracted with cost 6430731:
(Agg "j" (Prod
  (Matrix "A" "i" "j")
  (Agg "k" (Prod
    (Matrix "B" "j" "k")
    (Matrix "C" "k" "l")))))
```

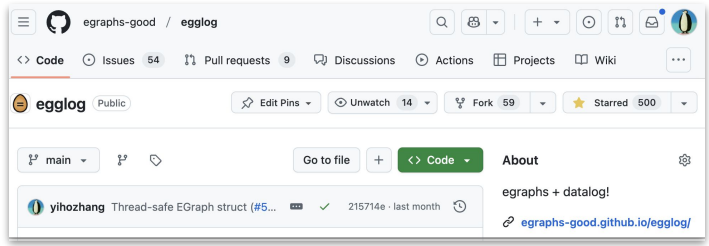


This talk

- EqSat: a promising approach to search-based program optimization
- EqSat ⊆ the Chase
- Cascades/Volcano ⊆ Equality Saturation
- EqSat unifies rule- and cost-based program optimization.



egraphs-good.github.io/egglog



github.com/egraphs-good/egglog