EqSat is not better than term rewriting*

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* but tree automata completion is!

Agenda

We study the termination problem of EqSat ($\text{TERM}_{\text{EqSat}}$):

- $\text{TERM}_{\text{EqSat}}$ and $\text{TERM}_{\text{TmRw}}$ do not imply each other.
    - This refutes the misconception that EqSat is always a better replacement of term rewriting.
- We show **tree automata completion (TAC)**, a technique similar to EqSat with the property that $\text{TERM}_{\text{TmRw}}$ implies TERM_{TAC}.
    - We show an application of Tree Automata completion to rewrite rule synthesis.
- We introduce two tricks for ensuring EqSat termination in practice and their corresponding guarantees.

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We study the termination problem of EqSat ($\mathbf{TERM}_{\mathbf{EqSat}}$):

Termination of Equality Saturation

Tree Automata Completion

Termination Tricks with Guarantees

The termination problem of EqSat

Definition

- Instance: a rewriting system R, a term t.
- Question: does running EqSat with R on initial term t terminate in a finite number of iterations?

If EqSat terminates, the equalities are saturated.

- Optimality in program optimization.
- Decidability in theory solving.

The termination problem of EqSat

Folklore: “EqSat is term rewriting but more powerful”.

- E-graphs can represent infinitely many terms; so
- EqSat should terminate for more term rewriting systems.

This is not true.

Associativity does not terminate

Let R be

$$(x \cdot y) \cdot z \to x \cdot (y \cdot z)$$
$$0 \cdot a \to 0$$

R is terminating in TmRw.

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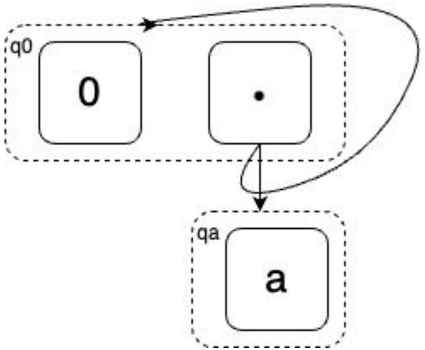
However, with initial term $0 \cdot a$, EqSat will apply rule $0 \cdot a \to 0$ and create a cyclic E-graph that represents

$$0,\ 0 \cdot a,\ (0 \cdot a) \cdot a,\ ((0 \cdot a) \cdot a) \cdot a,\ \ldots$$



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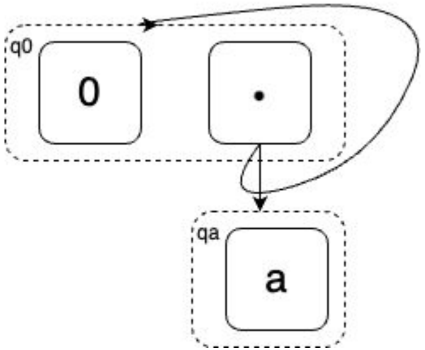
0, 0 · a, (0 · a) · a, ((0 · a) · a) · a, ...

Associativity will reassociate these terms and produce a, a · a, a · a · a, ..., which are pairwise inequivalent.



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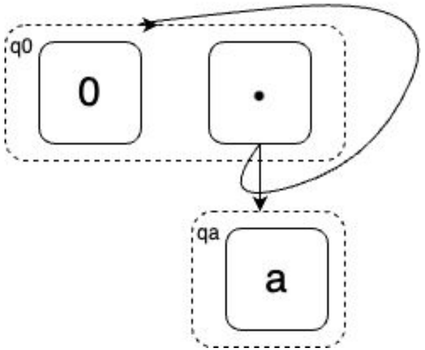


Associativity will reassociate these terms and produce a, a · a, a · a · a, ..., which are pairwise inequivalent.

This requires an infinite number of E-classes, which is impossible.

Convergent TRSes do not terminate

There are also convergent term rewriting systems that do not terminate in EqSat.

Convergence (termination + confluence) is one of the strongest properties in term rewriting.

Why?

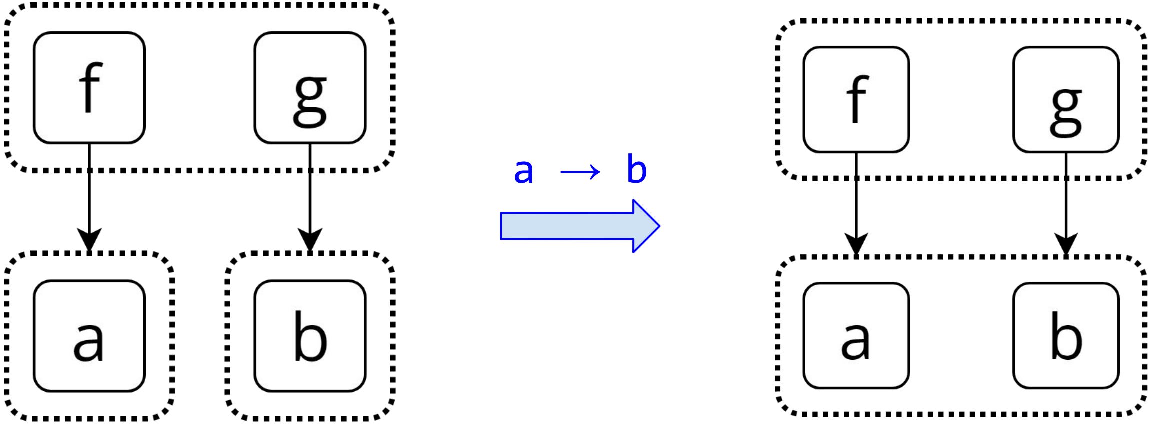
If a TRS is terminating, then the set of derivable terms should always be finite.

- It is natural to think EqSat should terminate as well.

The issue: **EqSat is not exactly term rewriting!**

- EqSat tracks equivalences, not rewritability!
- EqSat can derive terms not derivable in term rewriting.

Why?



Represents
f(a)   g(b)

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f(b)   g(a)

Term rewriting will never derive g(a)!

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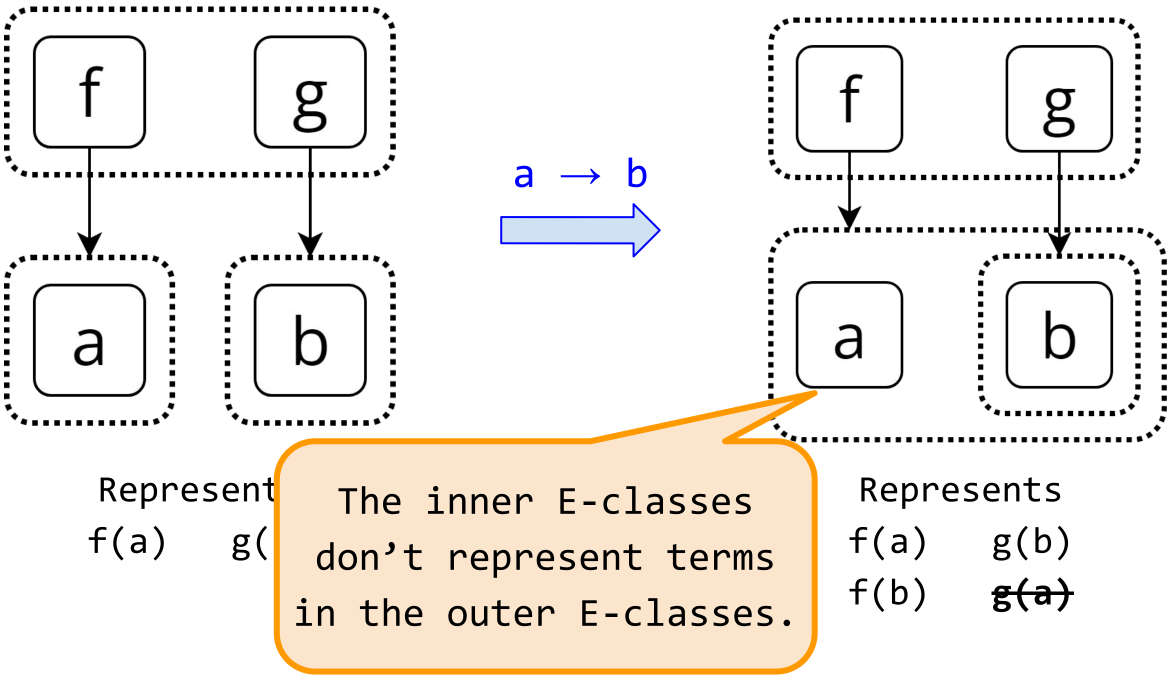
Termination of Equality Saturation

⇨ **Tree Automata Completion** ⇦

Termination Tricks with Guarantees

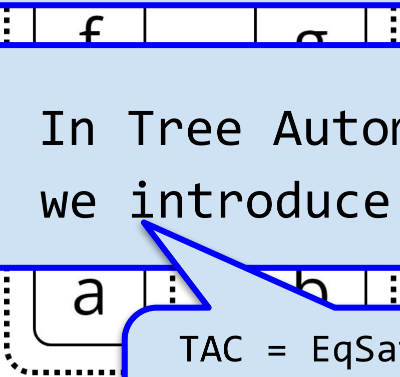
Tree automata completion



Tree

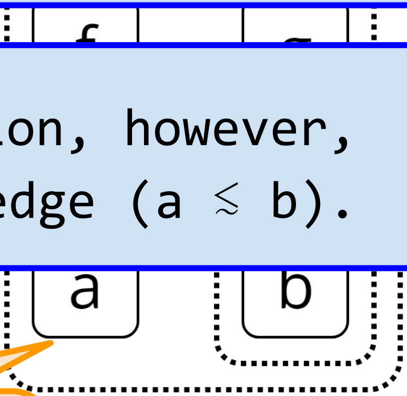
In EqSat, when we merge two terms, we introduce an equivalence edge between them (a ≈ b).

In Tree Automata Completion, however, we introduce an *ordered* edge (a ≲ b).



TAC = EqSat
- equivalence
+ preorder

...lasses
don't represent terms
in the outer E-classes.



Represents
f(a) g(b)
f(b) ~~g(a)~~

Tree

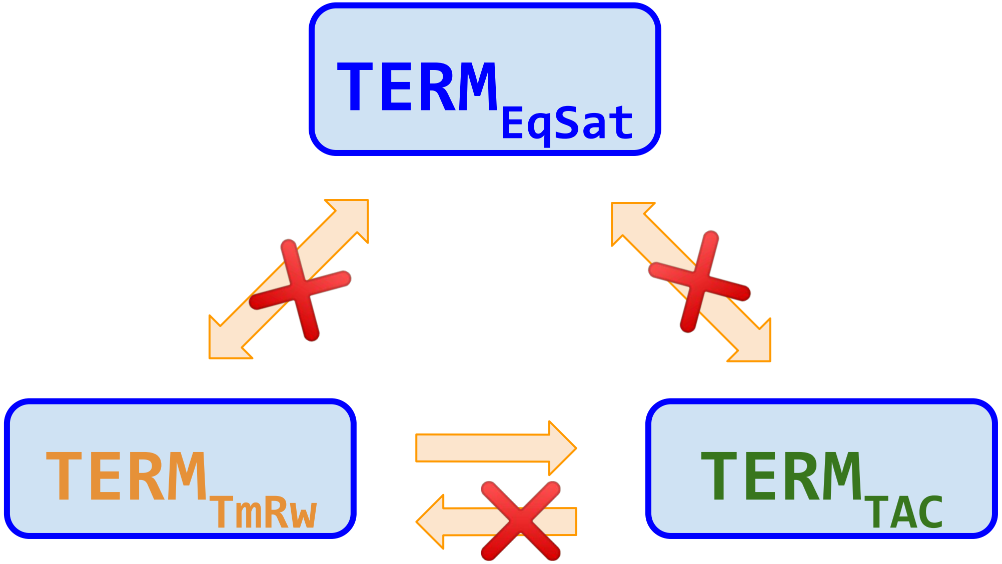
In EqSat, when we merge two terms, we introduce an equivalence edge between them (a ≈ b).

In Tree Automata Completion, however, we introduce an *ordered* edge (a ≲ b).

Guarantee: Tree Automata Completion will only derive terms derivable in term rewriting.

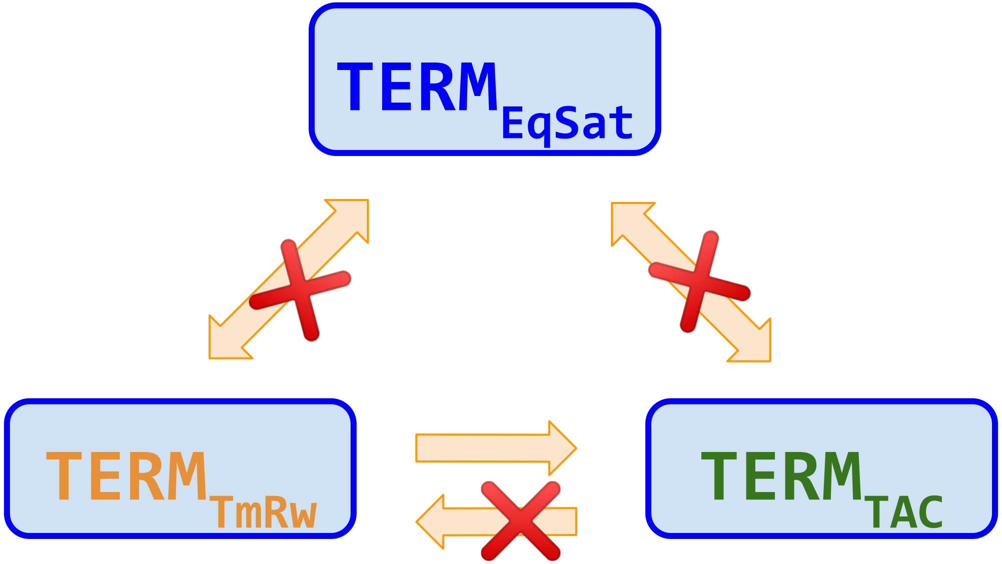
Corollary: $\text{TERM}_{\text{TmRw}}$ implies TERM_{TAC}.

Comparison



$TERM_{TmRw}$ and $TERM_{TAC}$ are known to be undecidable.

Comparison



$TERM_{TmRw}$ and $TERM_{TAC}$ are known to be undecidable.

We additionally showed $TERM_{EqSat}$ is undecidable.

Implementation of Tree Automata Completion

- We have not implemented it.
- Tree Automata Completion is computationally harder than EqSat.
    - Need to maintain and query a DAG than an equivalence relation.

Potential Applications of Tree Automata Completion

- Applications that require strong **termination guarantees.**
- Applications where **rewritability/refinement** is desired.
- Ruler: rewrite rule synthesis.

This talk

Tree automata completion for rewrite rule synthesis

Ruler's rule validation problem

Instance: a list of expression pairs (lhs_i, rhs_i)

Question: For each i, can EqSat use lhs_i to derive rhs_i?

Ruler currently inserts all lhs_i, runs EqSat once, and checks if each rhs_i exists and is equivalent to lhs_i.

- \+ Very fast thanks to batching.
- \- This can be unsound.

Tree automata completion for rewrite rule synthesis

x + 0 → x

x × 1 → x

(a × 1) + 0 ⟹? a

a × 1 ⟹? a + 0

Tree automata completion for rewrite rule synthesis

```
x + 0 → x
x × 1 → x
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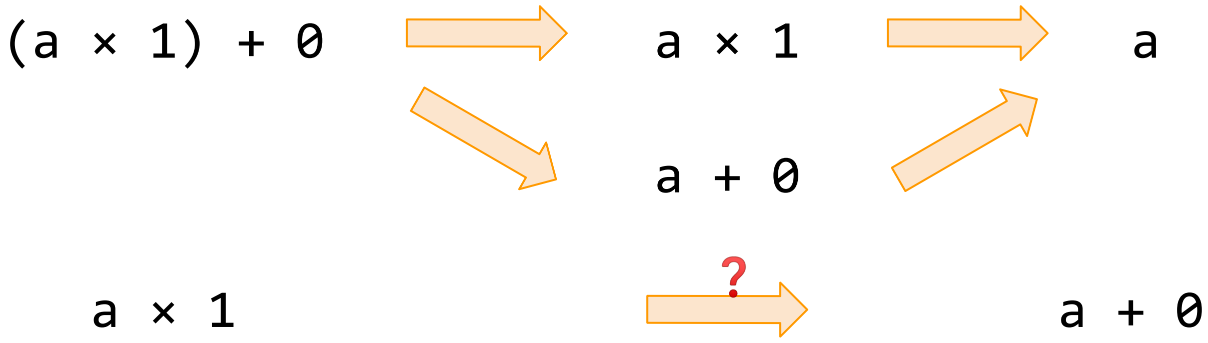

Tree automata completion for rewrite rule synthesis

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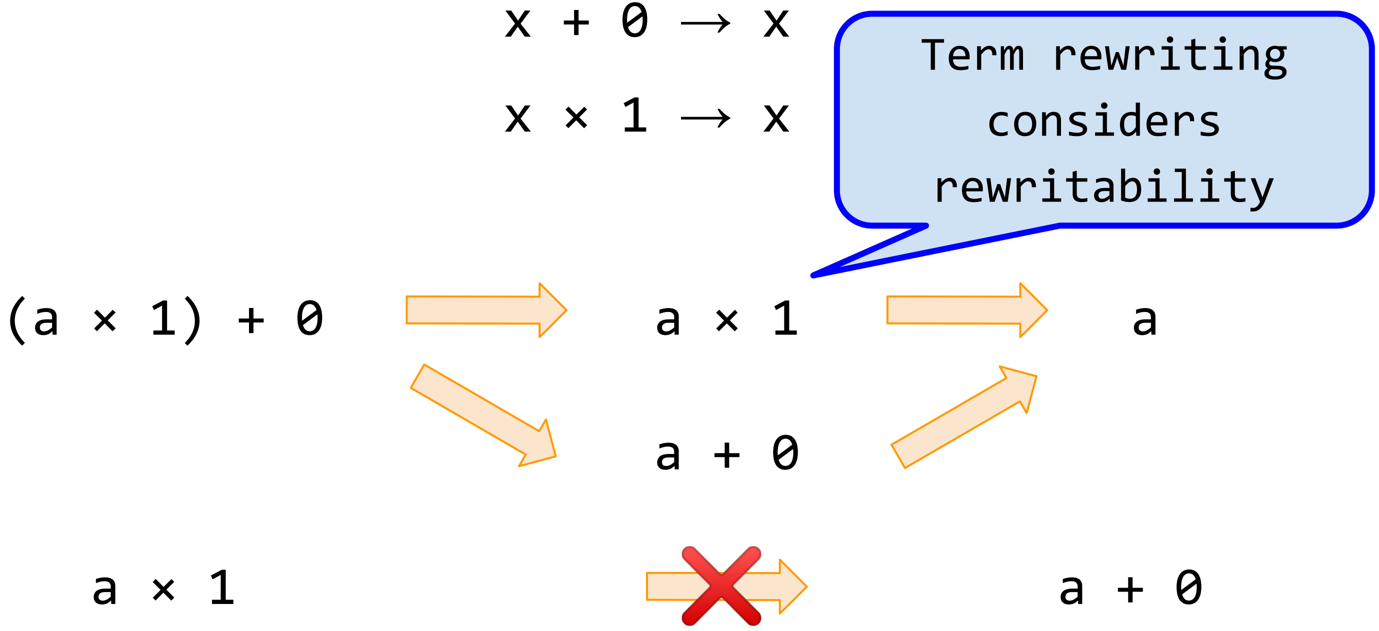

Tree automata completion for rewrite rule synthesis

x + 0 → x

x × 1 → x

EqSat considers term equivalences

(a × 1) + 0 $\stackrel{?}{=}$ a

a × 1 $\stackrel{?}{=}$ a + 0

Tree automata completion for rewrite rule synthesis

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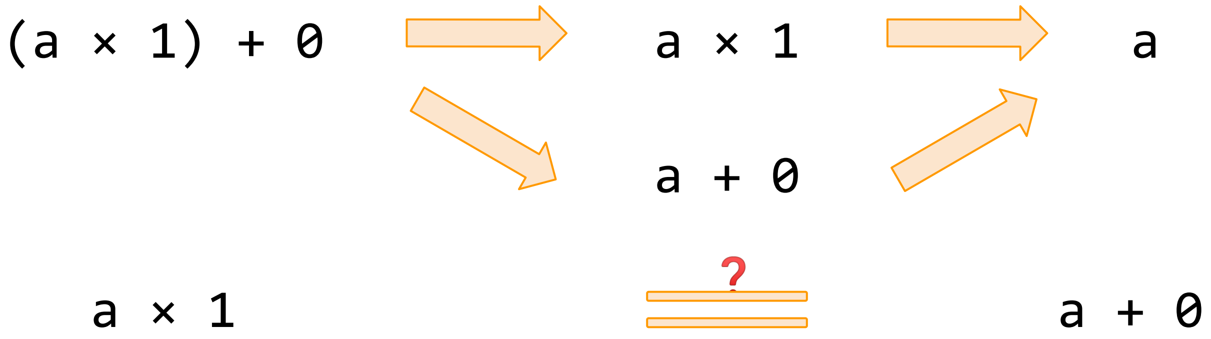

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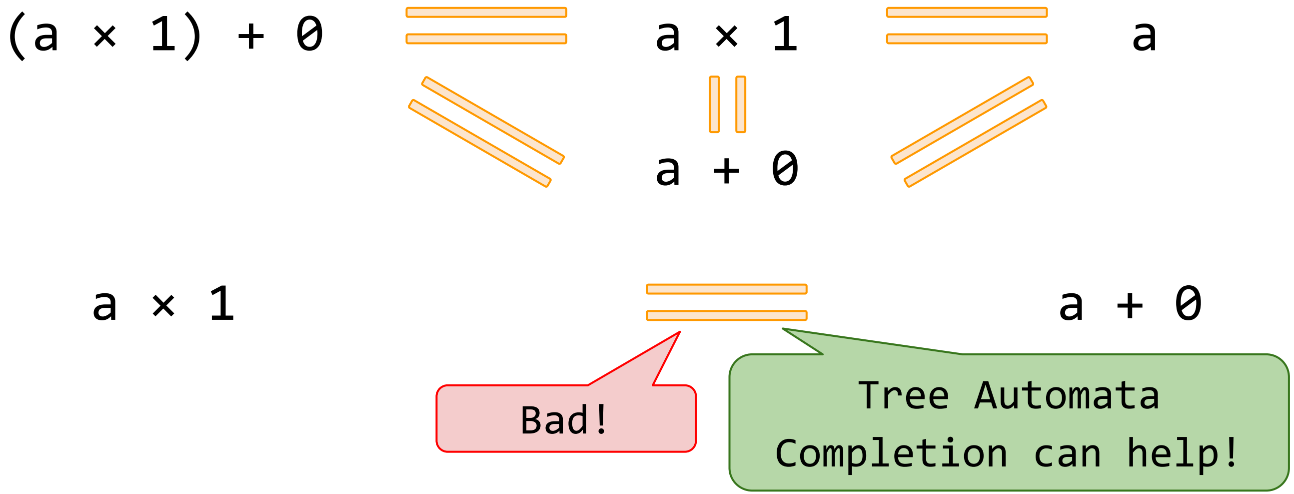

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Practical approaches to termination

In practice, people ensure termination by running EqSat for a finite number of iterations.

Guarantee: If there exists a proof to u = v of the form

$$\exists\ w.\ u \longrightarrow^{\leq N} w \longleftarrow^{\leq N} v,$$

running EqSat for N iterations can prove u = v.

Observation: Different termination strategy gives us different guarantees.

Merge-only Equality Saturation

Given an E-graph G, merge-only EqSat applies a rule only when both the left-hand side and the right-hand side are already in the E-graph.

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Termination: Notice merge-only EqSat shrinks the number of E-classes in each iteration.

Merge-only Equality Saturation

Given an E-graph G, merge-only EqSat applies a rule only when both the left-hand side and the right-hand side are already in the E-graph.

Termination: Notice merge-only EqSat shrinks the number of E-classes in each iteration.

Guarantee: If u = v can be proved using only terms in G, this proof can be obtained with merge-only EqSat.

(Depth-)bound … ation

Min depth of all terms represented.

Still admits infinite terms!

Depth-bounded EqSat tracks ca … class … s:

$$\texttt{depth(c) = min}_{\texttt{c represents t}}\texttt{depth(t)}$$

(Depth-)bounded Equality Saturation

Depth-bounded EqSat tracks each E-class with a depth analysis:

$$\text{depth}(c) = \min_{c \text{ represents } t} \text{depth}(t)$$

During rule application, we apply a rule only when the right-hand side has depth ≤ N.

(Depth-)bounded Equality Saturation

Depth-bounded EqSat tracks each E-class with a depth analysis:

$$\text{depth}(c) = \min_{c \text{ represents } t} \text{depth}(t)$$

During rule application, we apply a rule only when the right-hand side has depth ≤ N.

Termination: The depth constraint bounds the # of possible E-classes, which bounds the # of possible E-graphs.

(Depth-)bounded Equality Saturation

Guarantee: if u = v can be proved using terms with depth ≤ N,
this proof can be obtained with depth-bounded EqSat.

+ Useful for program optimization.
- Hardly terminate for realistic N.

Can be generalized to *F*-bounded EqSat, where *F* is some constraints that bounds the number of possible E-classes (e.g., size).

Takeaways

- The termination problem of EqSat is not trivial.
- Tree Automata Completion = EqSat - ($\approx$) + ($\lesssim$).

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- The termination problem of EqSat is not trivial.
- Tree Automata Completion = EqSat - ($\approx$) + ($\lesssim$).
- Two termination strategies.
  - Merge-only EqSat.
  - Depth-bounded EqSat.