Better together: Unifying Datalog and Equality Saturation

Yihong Zhang, Remy Wang, Oliver Flatt, David Cao,
Philip Zucker, Eli Rosenthal, Zach Tatlock, Max Willsey

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Problem: we want everything

In Term Rewriting with EqSat

+ Fast equational reasoning
- Poor analysis support

In Program Analysis with Datalog

+ Composable program analyses
- Quadratic equational reasoning

Can we build one system that subsumes both?

Yes! But How?!

To unify Datalog and EqSat, all you need are

- Functional dependency.
- Functional dependency repair.

Background

EqSat: term rewriting with e-graphs

Big data systems

- Tensor programs [MLSys ‘21, MAPS ‘21]
- Sparse linear algebra [VLDB ‘20]
- Recursive queries [SIGMOD ‘22]

Hardware

- DSP vectorization [ASPLOS ‘23]
- Datapath optimization [ASP-DAC ‘23]

Program optimization

- Imperative programs [POPL ‘09]
- Functional programs [EGRAPHS ‘22]
- Floating-point expression [PLDI ‘15]

Program synthesis

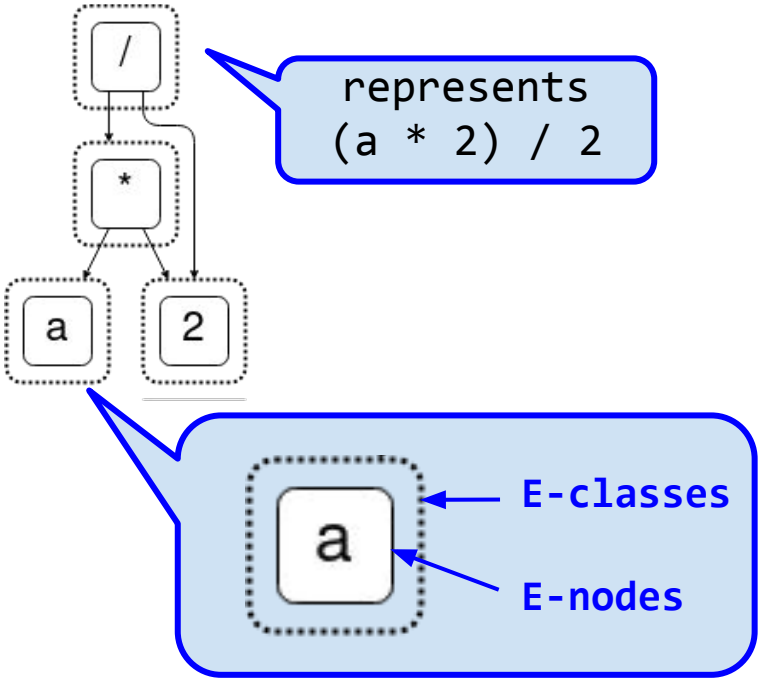
- CAD parametrization [PLDI ‘20]
- Rewrite rule synthesis [OOPSLA ‘21]

RisingLight

An Educational OLAP Database System

BYTECODE ALLIANCE

E-graphs and Equality saturation



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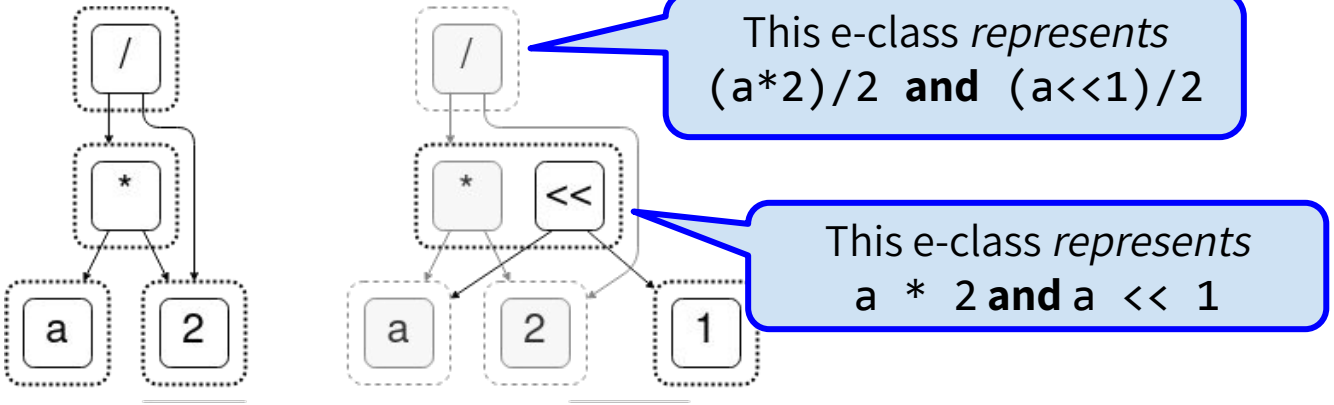
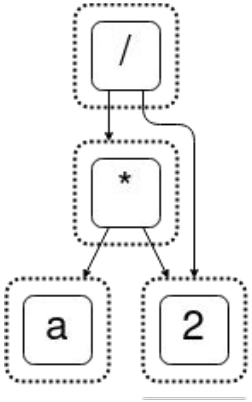
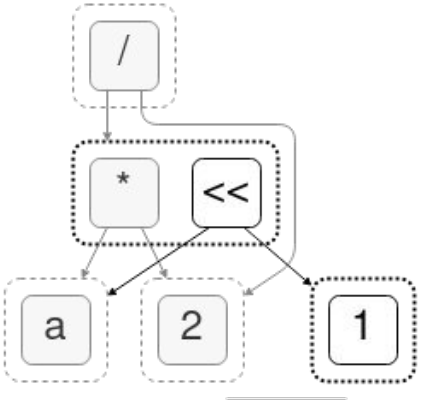



```
x * 2 => x << 1
```

E-graphs and Equality saturation





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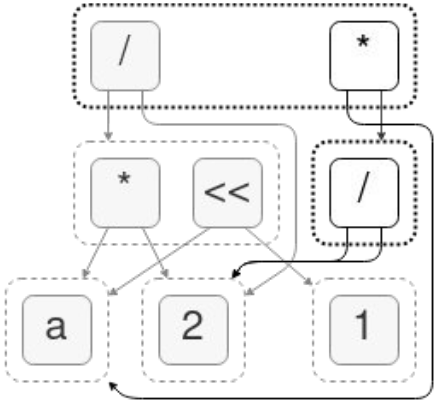
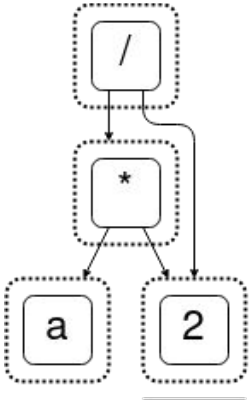
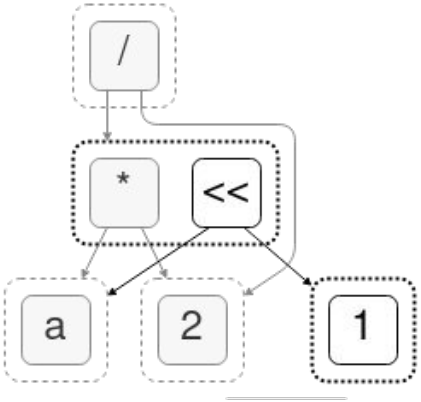


`(x * y) / z => x * (y / z)`

E-graphs and Equality saturation





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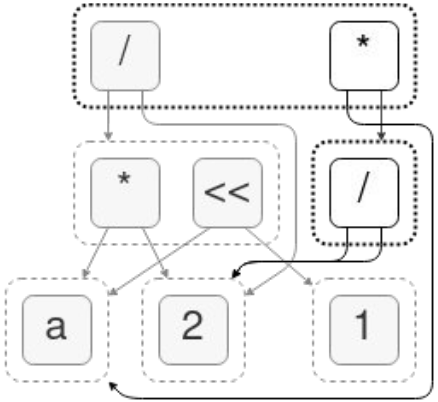

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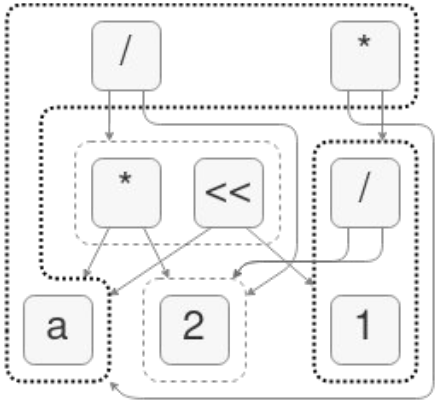


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x / x => 1
x * 1 => x
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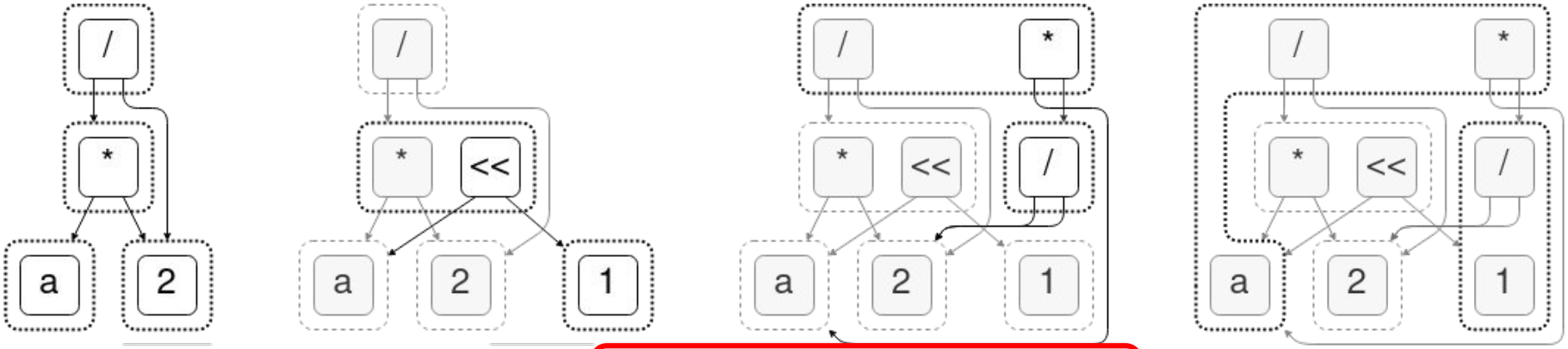
loop until
fixpoint / timeout!

E-graphs and Equality saturation



x * 2 => x <<

x has to be non-zero!

x / x => 1

x * 1 => x

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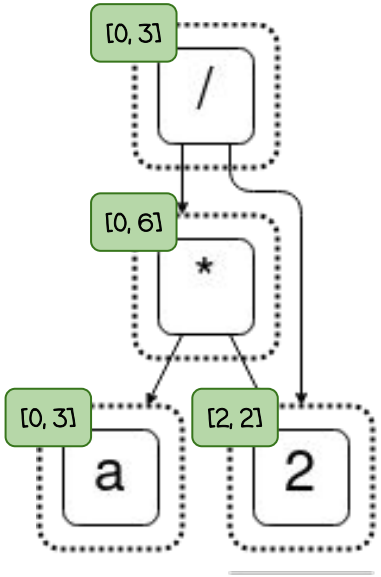
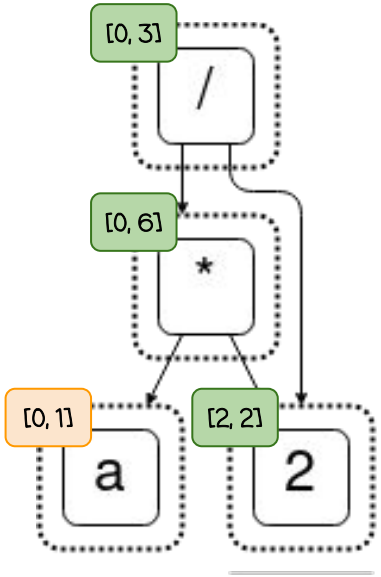
Needs semantic
information here

E-class analyses



- Semantic analyses over E-graphs
- Each E-class is abstracted w/ a lattice.

E-class analyses



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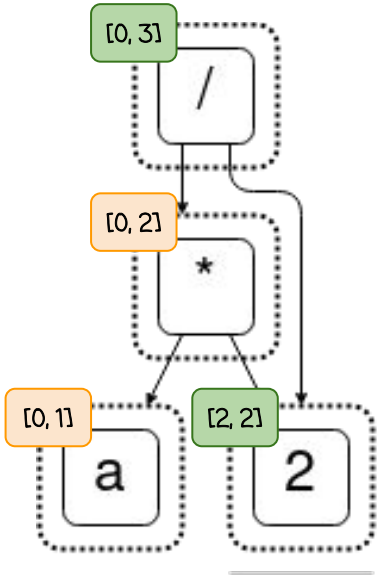

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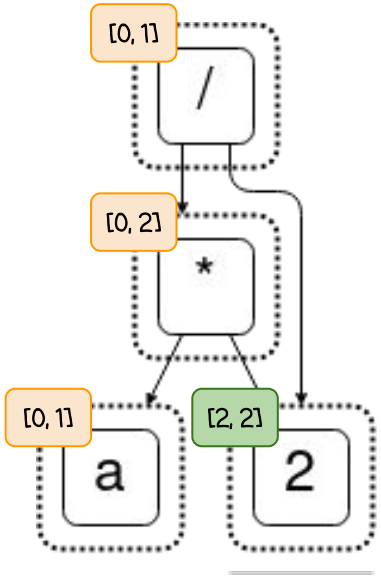

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E-class analyses

[0,1]

E-class analysis has severe limitations:

- Only one analysis allowed.
- Facts only propagate from children to parents.
  - Type checking 😭
- Monolithic Rust implementation of one big analysis.
  - Not composable!

tice.

BUT: analyses are rules, too!

Program analysis in Datalog

- Multiple analyses ✅
- Modular ✅
- Composable ✅

```
// If expression e is a number,
// its lower bound is itself
num(n, e) ⇒ lower_bound(e, n).

// If expression e has the form x + y,
// its lower bound is the lower bound of x
// plus the lower bound of y.
add(x, y, e) ∧
  lower_bound(x, lx) ∧
  lower_bound(y, ly) ⇒
lower_bound(e, lx + ly).

// If the lower bound of e is greater than 0,
// e is nonzero.
lower_bound(e, le) ∧ le > 0 ⇒
  nonzero(e)
```

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Pr

Can we do EqSat in Datalog?

We need to express in Datalog

$$(+\ (+\ x\ y)\ z) \longrightarrow (+\ x\ (+\ y\ z))$$

triggers

actions

Relational E-matching (POPL 2022)

- E-matching: pattern matching over the e-graph (triggers)

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- Significant speedups.
- This handles triggers ✅
- *What about actions?*

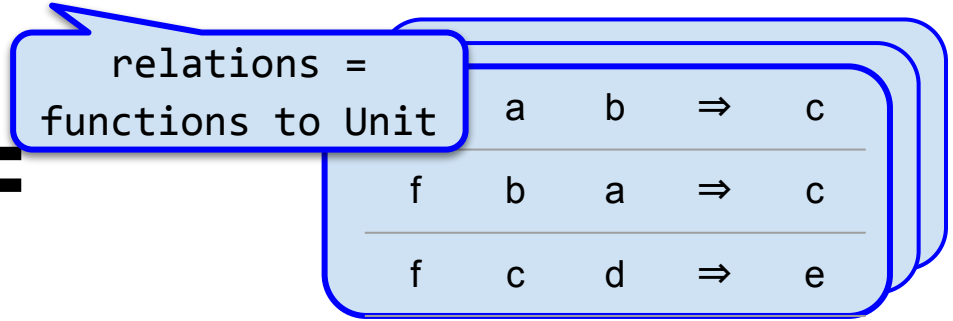
`egglog`: unifying Datalog and EqSat

egglog's key concept: functions

Using a function-first database design

Database

=



Set of function tables

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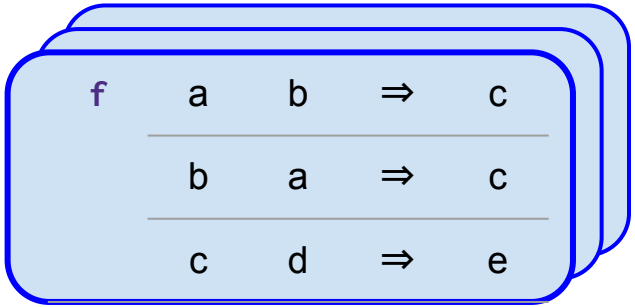

Set of function tables

Now we can talk about

- terms like `f(f(a, b), d)` and
- equivalences like `f(a, b)` = `f(b, a)`

Equality saturation in egglog

```
(datatype Math (Num i64)
               (Var String)
               (Add Math Math)
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;; expr = 3 * (x + 2)
(define expr (Mul (Num 3) (Add (Var "x") (Num 2))))

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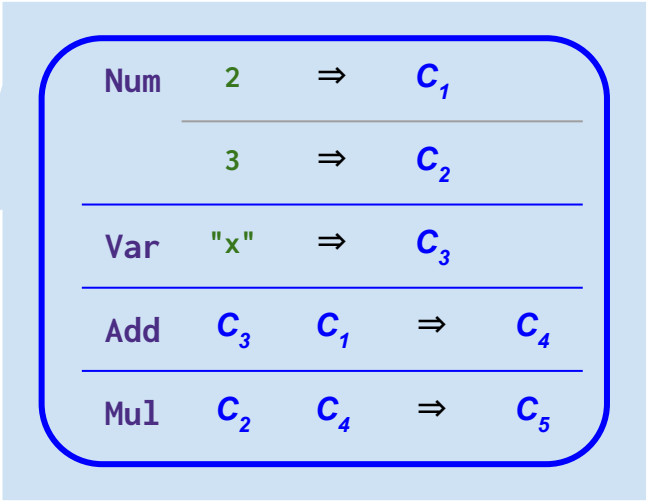

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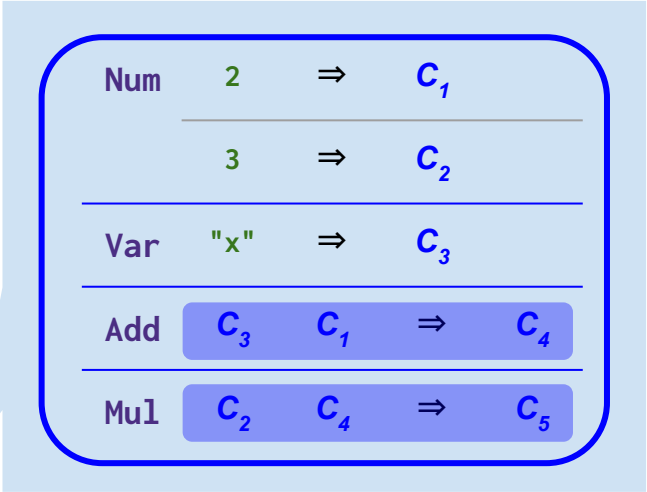

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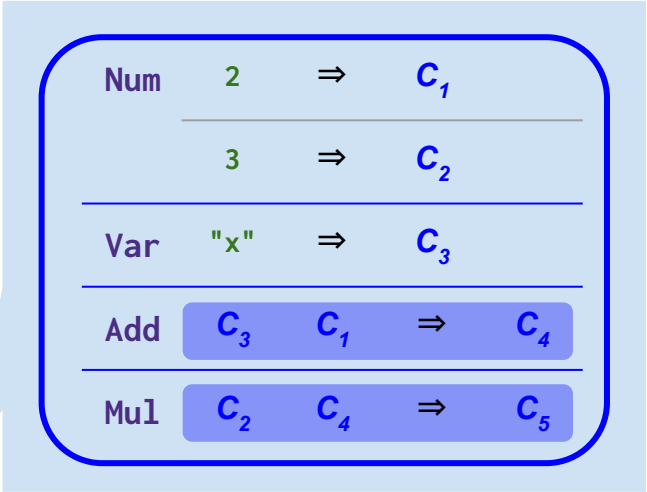

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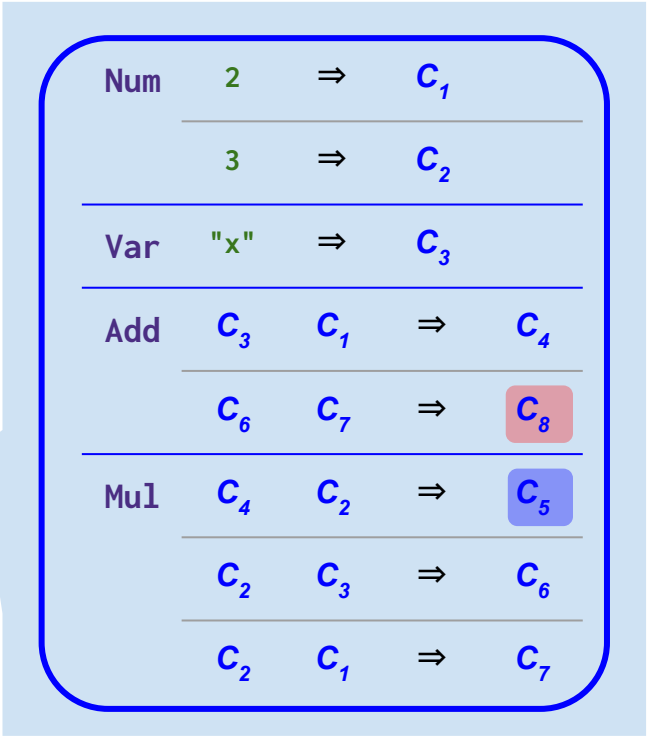

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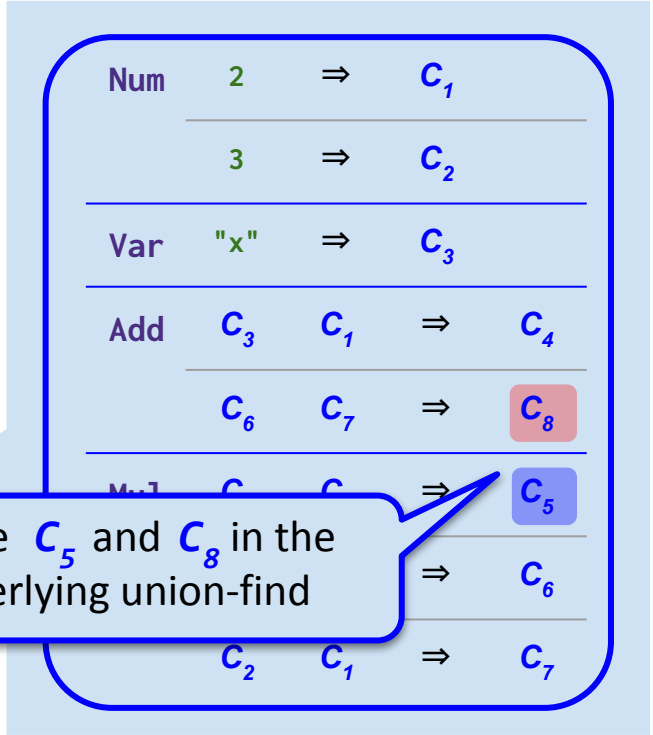
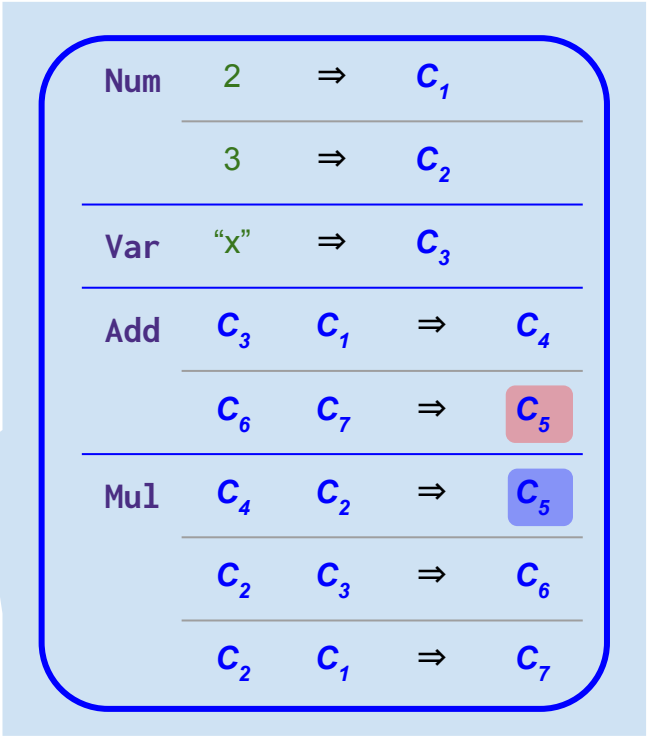


Equality saturation in egglog

```
(datatype Math (Num i64)
               (Var String)
               (Add Math Math)
               (Mul Math Math))

;; expr = 3 * (x + 2)
(define expr (Mul (Num 3) (Add (Var "x") (Num 2))))

;; x + y => y + x
(rewrite (Add x y)          (Add y x))
;; x * (y + z) => x * y + x * z
(rewrite (Mul x (Add y z)) (Add (Mul x y) (Mul x z)))
;; Num(x) + Num(y) => Num(x + y)
(rewrite (Add (Num x) (Num y)) (Num (+ x y)))
;; Num(x) * Num(y) => Num(x * y)
(rewrite (Mul (Num x) (Num y)) (Num (* x y)))
```



Key idea: functional dependency repair

- Func’s args should uniquely determine the output.
- This is what makes a function a function.
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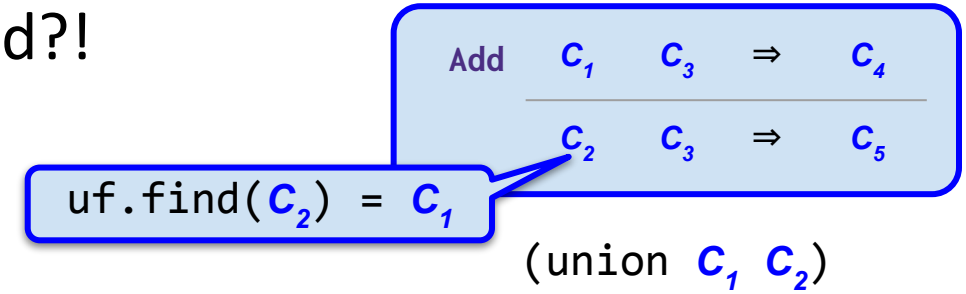

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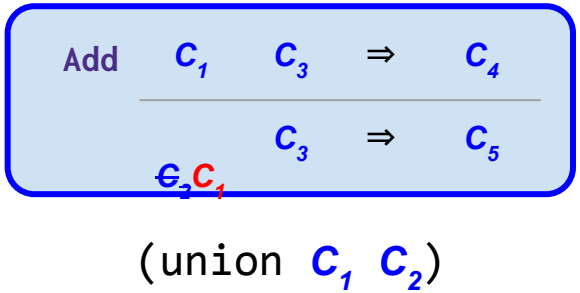

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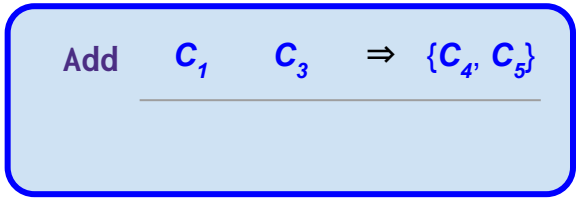

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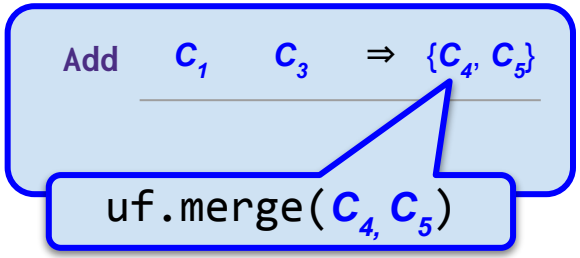

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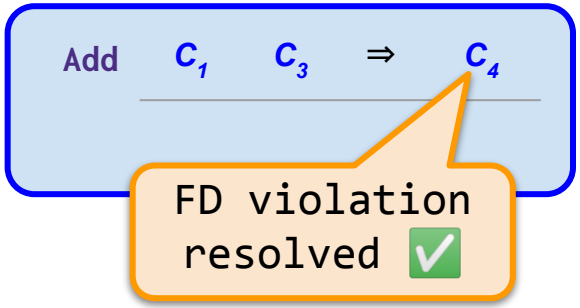
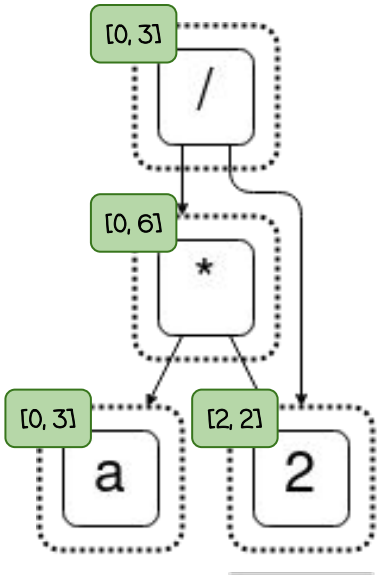


Key idea: functional dependency repair

The same mechanism also enables **composable** analyses.

```
(function hi (Math) Rational)

(function lo (Math) Rational)
```



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(function hi (Math) Rational :merge (min old new))

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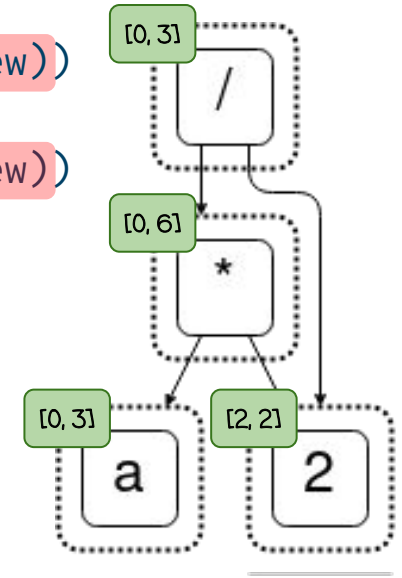

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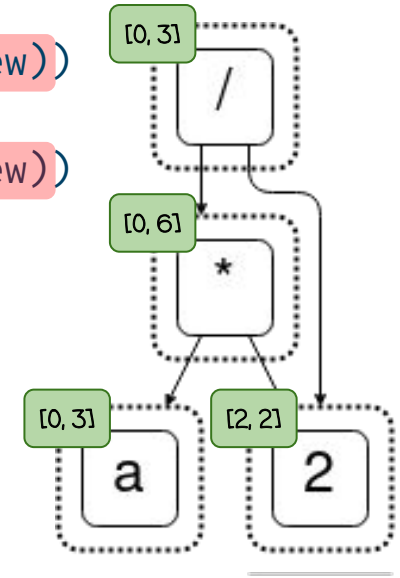
hi	c_1	⇒	3
	c_2	⇒	2

(union c_1 c_2)



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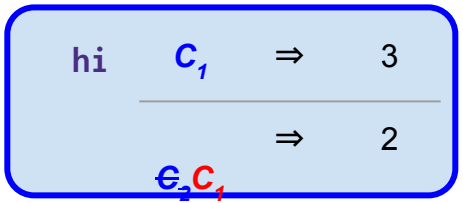

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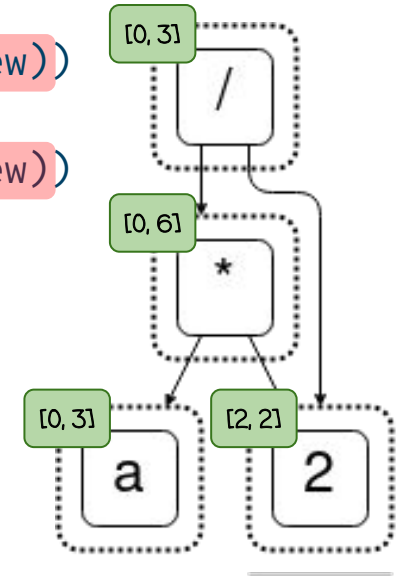

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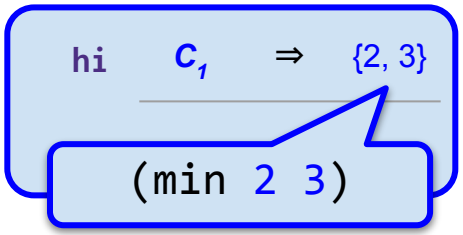

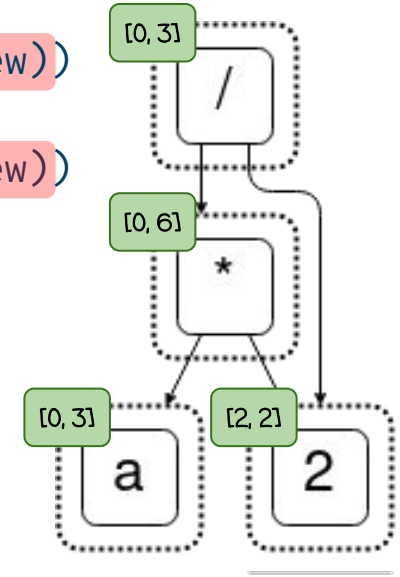

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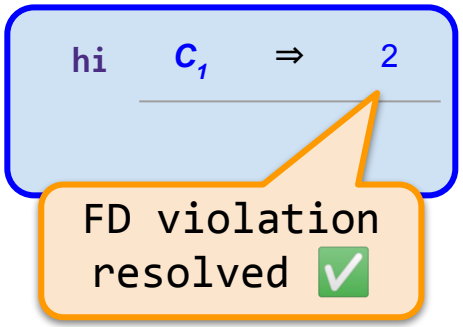

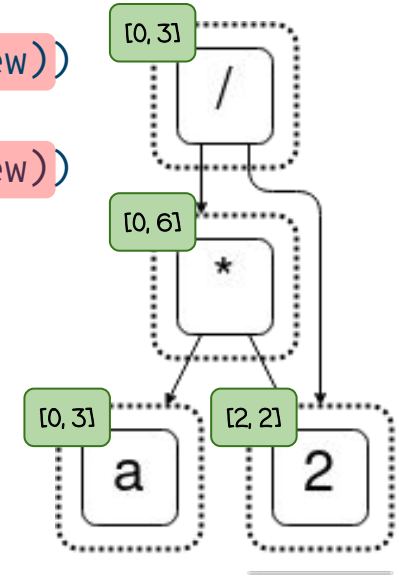

Evaluation

Enabling new optimizations

Database-like architecture

- Relational e-matching.
- Efficient query evaluation.

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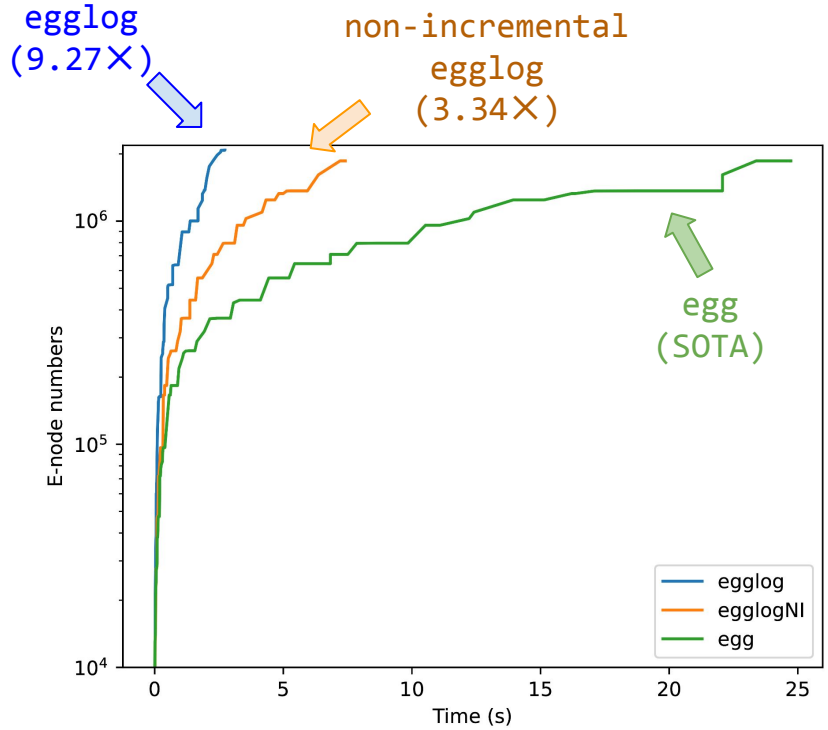
Incrementalization

- Incremental EqSat is hard.
- We use the standard semi-naive evaluation of Datalog to make EqSat incremental for free.



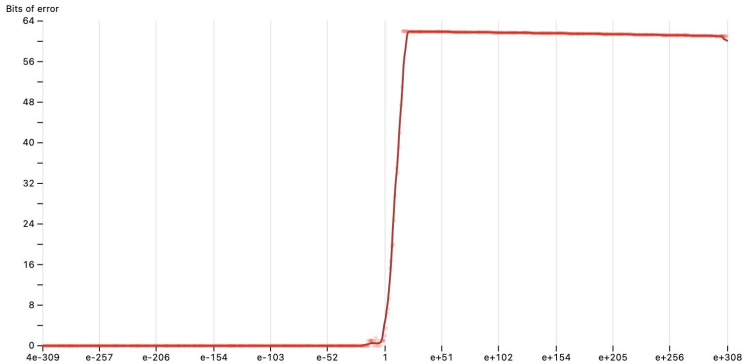
`math` micro-benchmark

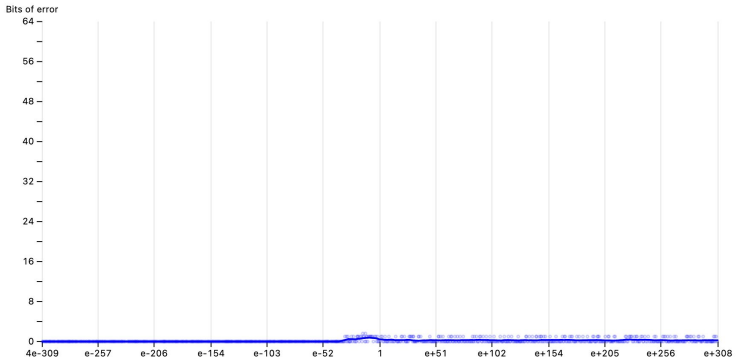
Herbie

$$\sqrt{x+1}-\sqrt{x}$$

$$\frac{1}{\sqrt{1+x}+\sqrt{x}}$$

less floating-point errors,
more accurate!

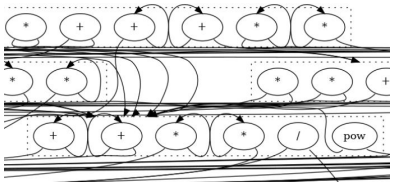




Herbie

$$\sqrt{x+1} - \sqrt{x}$$

EqSat



Extract

$$\frac{1}{\sqrt{1+x} + \sqrt{x}}$$

$x + y \Rightarrow y + x$
$(x + y) + z \Rightarrow x + (y + z)$
$\sin^2 x + \cos^2 x \Rightarrow 1$
$x^{yz} \Rightarrow x^y \cdot x^z$
$x^{-1} \Rightarrow 1 / x$
…

Not all identities can be expressed as syntactic rules.

$x / x \Rightarrow 1$ **when** $x \neq 0$
$(x \cdot y) / z \Rightarrow x / (z / y)$
when $y \neq 0$
$x - y \Rightarrow (x^3 - y^3) / (x^2 + xy + y^2)$
when $x \neq 0$ **or** $y \neq 0$
…

Herbie

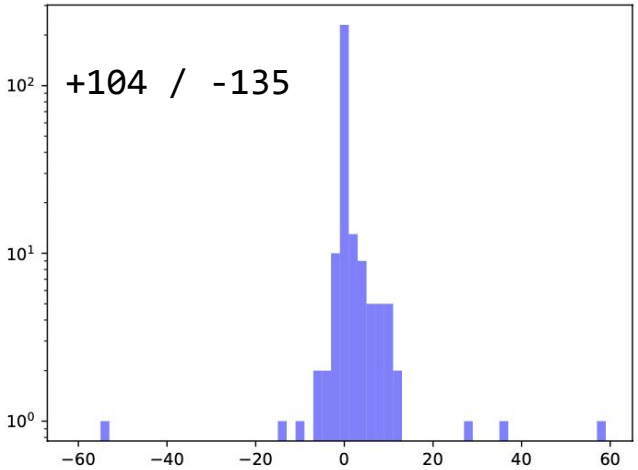
When unsoundness is detected, Herbie has to discard the results and roll back 😭

We make Herbie's rules sound with E-graph program analyses in egglog:

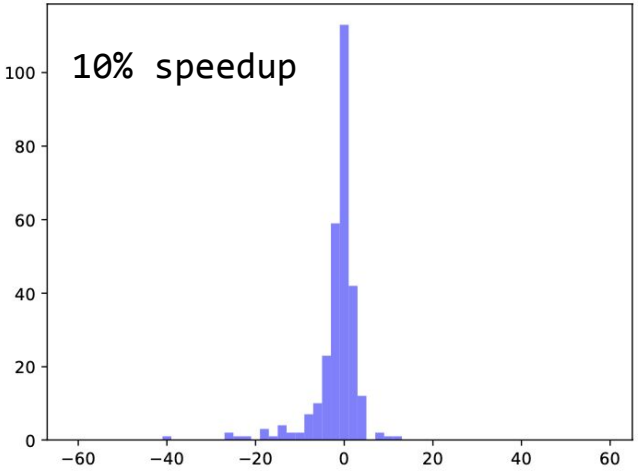
- Interval analysis
- Definability analysis

y^2)

Results on Herbie’s benchmark



Accuracy (“# bits”)



Time (s)

Results on Herbie's benchmark

Our reimplementation in egglog achieves a comparable accuracy and performance, but does not suffer from the soundness issue in original Herbie.

Herbie's design made simpler ✅

Bringing the power of unification to Datalog

Datalog is good at program reasoning tasks such as

- Pointer analyses.
- Type checking/inference

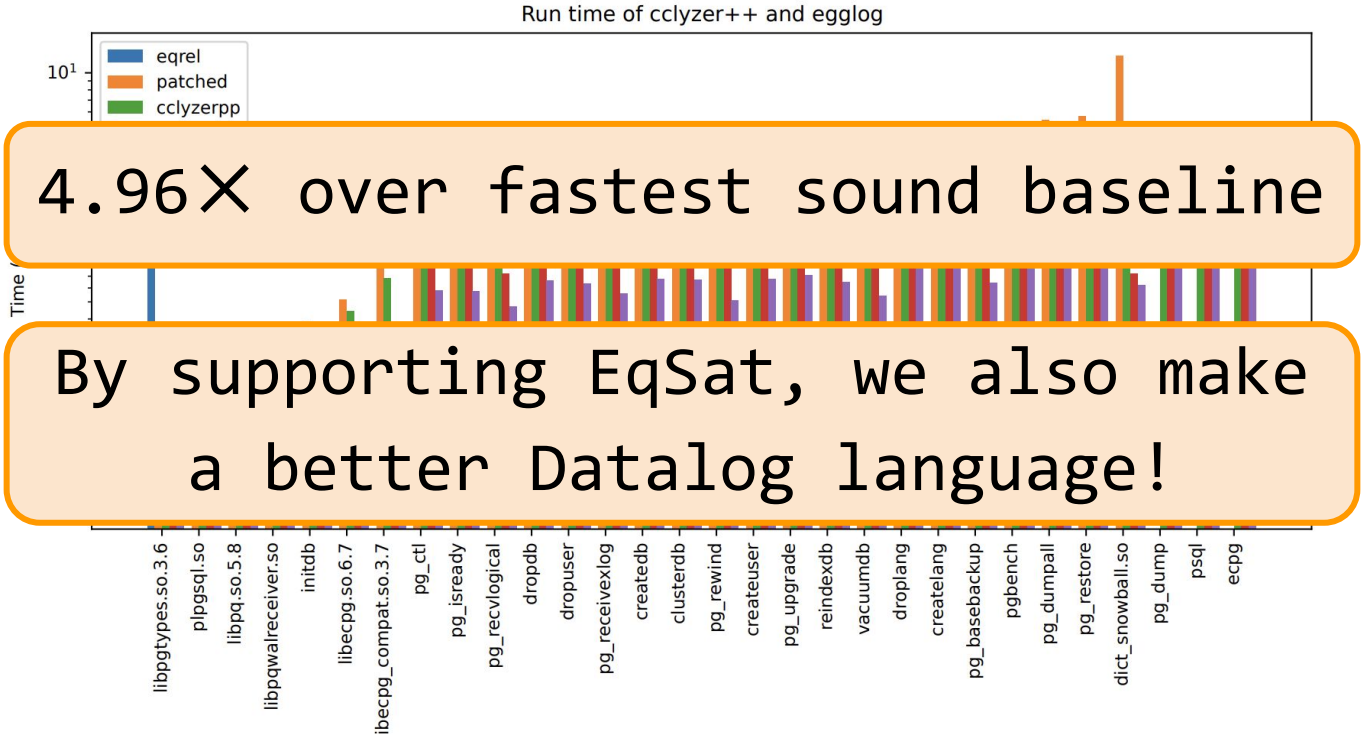
However, many advanced program reasoning tasks also require equivalence reasoning

- Steensgaard pointer analysis.
- Hindley-Milner type inference.

Datalog: 😟

egglog: ☺️

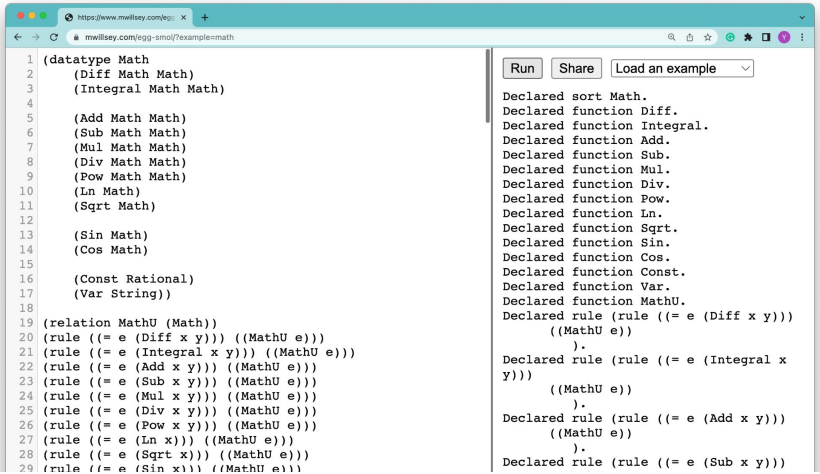
Steensgaard-style points-to analysis



egglog: Unifying Datalog and Equality Saturation

By unifying Datalog and EqSat, we get

- ✅ Fast equational reasoning *a la* EqSat.
- ✅ Rich composable analyses *a la* Datalog.
- ✅ Fast and incremental eval with DB magic.
- ✅ User-friendly language interface.



egraphs-good.github.io/egglog

Thank you

Remy Wang

Oliver Flatt

David Cao

Philip Zucker

Eli Rosenthal

Zach Tatlock

Max Willsey

Key idea: functional dependency repair

The s…yses.